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Outcomes with Assessment Standards

for

Pure Mathematics 10

August 2002

This document is intended to assist teachers with the provincial implementation of Pure Mathematics 10.

This document replaces the February 2000 and September 2000 interim drafts and reflects the 2002 Program of Studies.

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The primary intended audience for this document is:

Administrators	
Counsellors	
General Audience	
Parent School Councils	
Parents	
Students	
Teachers	✓

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INTRODUCTION

Pure Mathematics 10 was provincially implemented in September 1998. Teachers implementing this new course expressed a need for a common understanding of the curriculum and assessment standards. In response to this need, and in keeping with Alberta Learning's goal of establishing and effectively communicating clear outcomes and high standards for each area of learning, this standards document was developed.

PURPOSE

Outcomes with Assessment Standards for Pure Mathematics 10 contains samples of tasks linked to the specific outcomes from the program of studies, along with information and commentaries about standards. Its purpose is to provide teachers of Pure Mathematics 10 with clearly stated standards to use as guidelines in their classroom instruction and assessment practices. This document is not intended as an assessment tool or test package, but only as a guide that provides models of tasks that can be assigned or discussed in the classroom. The sample tasks in this document are intended for teacher use, but also can be used to communicate to the broader educational community examples of acceptable student work and excellent student work in Pure Mathematics 10.

DEFINITIONS AND TERMINOLOGY

Standards

A *standard* is a reference point used in planning and evaluation. In evaluating educational performance, the following standards apply:

- *curriculum* and *assessment standards* apply to the assessment of individual students
- *achievement standards* apply to the assessment of student populations.

In this document, only curriculum and assessment standards are discussed.

Curriculum Standards

Curriculum standards are outcomes for a course or grade level of a program. The curriculum standards for Pure Mathematics 10 are defined by the general and specific outcomes outlined in the program of studies.

Outcomes

General outcomes are concise statements identifying what it is that students are expected to know and be able to do upon completion of a course or grade level of a program.

Specific outcomes are statements identifying the component knowledge, skills and attitudes of a general outcome. Specific outcomes identify a range of contexts in which the general outcomes apply.

Assessment Standards

Assessment standards are the criteria used for judging individual student achievement relative to the curriculum standards.

Assessment Instrument

An *assessment instrument* is a group of questions or tasks given to students to ascertain if they have met the requirements of the acceptable standard or the standard of excellence.

Blueprint

A *blueprint* is a classification of the questions or tasks making up a particular assessment instrument or an assessment plan. The classification may be in terms of standards (acceptable or excellent), content sections (topics or titles), mathematical understandings (concepts, procedures, problem-solving skills), or any other such classification.

Conceptual Understanding

Tasks assessing *conceptual understanding* are characterized by the use of such action verbs as defining, demonstrating, describing, developing, establishing, explaining, giving, identifying, illustrating, linking, recognizing and representing.

Procedural Knowledge

Tasks assessing *procedural knowledge* are characterized by the use of such action verbs as approximating, calculating, constructing, estimating, factoring, locating, rationalizing, simplifying, sketching, solving, using and verifying.

Problem-solving Skills

Problem-solving skills refer to the construction of models describing applications in a broad range of contexts. They refer to the combination of algorithms to determine more complicated solutions and the use of appropriate technology to approximate difficult models or to carry out long calculation procedures. They also refer to the investigation of phenomena, the collection and interpretation of data in various contexts, and the transfer of knowledge from mathematics to other areas of human endeavour. Problem-solving tasks are characterized by the use of such action verbs as adapting, analyzing, combining, communicating, comparing, connecting, constructing, deriving, evaluating, fitting, investigating, justifying, modelling, proving, reconstructing and translating.

STANDARDS FOR PURE MATHEMATICS 10

The content specified in the program of studies for Pure Mathematics 10 contains the knowledge, skills and attitudes that all students are expected to acquire. As well, there are specific outcomes for problem solving and the use of technology. For complete details of the Pure Mathematics 10 course structure, refer to the program of studies.

Pure Mathematics 10 is designed to follow directly from Grade 9 Mathematics, so students taking Pure Mathematics 10 are presumed to have reached the acceptable standard or better in the outcomes of Grade 9 Mathematics. In particular, students will have facility in using the scientific calculator; have an ability to add, subtract and multiply polynomials; have an understanding of the Pythagorean theorem and the three primary trigonometric ratios; and have an ability to produce estimated lines of best fit for bivariate data.

The assessment standards for Pure Mathematics 10 include an acceptable and an excellent level of performance. Student performance should be measured on a range of tasks, some of which are routine and obvious tasks in familiar contexts, and others which are nonroutine tasks in unfamiliar contexts.

Acceptable Standard

The *acceptable standard* of achievement in Pure Mathematics 10 is met by students who receive a course mark between and including 50 per cent and 79 per cent. Typically, these students have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined in the Pure Mathematics 10 program of studies. These students can apply this knowledge to a limited range of familiar problem contexts.

Standard of Excellence

The *standard of excellence* for achievement in Pure Mathematics 10 is met by students who receive a course mark at or above 80 per cent. Typically, these students have gained a breadth and depth of knowledge regarding the concepts and procedures, as well as the ability to apply this knowledge to a broad range of familiar and unfamiliar problem contexts. This standard signifies high-quality performance relative to the general and specific outcomes in the Pure Mathematics 10 program of studies.

Description of Standards

The following statements describe what is expected of Pure Mathematics 10 students who meet the *acceptable standard* or the *standard of excellence* on independent work. The statements represent the standards against which student achievement is measured.

Acceptable Standard	*Standard of Excellence*
Students who meet the *acceptable standard* in Pure Mathematics 10 consistently perform acceptable work on routine and obvious tasks in familiar contexts.	Students who meet the *standard of excellence* in Pure Mathematics 10 consistently perform excellent work on routine and obvious tasks in familiar contexts, and acceptable work on nonroutine tasks in unfamiliar contexts.
These students have a basic understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to pure mathematics at the 10 level, and apply what they know in daily living contexts.	These students have a comprehensive understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to pure mathematics at the 10 level, apply what they know in daily living contexts, and provide alternative solution procedures to verify results.

To meet the *acceptable standard*, students communicate about mathematical situations in an understandable way, using appropriate everyday and mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar contexts; and they construct mathematical models.	To meet the *standard of excellence,* students communicate about mathematical situations in a clear way, using numbers, diagrams and appropriate mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar and unfamiliar contexts; and they construct mathematical models by translating words into suitable numbers, diagrams, tables, equations and variables.
Students meeting the *acceptable standard* apply what they know in solving straightforward problems in familiar settings and in analyzing simple mathematical models. They describe the steps they used to solve a particular problem, and verify and defend their solution to the problem.	Students meeting the *standard of excellence* apply what they know in solving routine and nonroutine problems in a broad range of settings. They describe the steps they used to solve a particular problem; defend their solution to the problem; and, where appropriate, provide alternative solution procedures to verify results.
Students meeting the *acceptable standard* have a positive attitude toward mathematics and a sense of personal competence in using mathematics. They demonstrate confidence when using common mathematical procedures and when applying problem-solving strategies in familiar settings.	Students meeting the *standard of excellence* have a positive attitude toward mathematics and show confidence in using mathematics meaningfully. They are self-motivated risk takers who persevere when solving novel problems. They take initiative in trying new methods and are creative in their approach to problem solving.

ASSESSMENT PLANS, SUGGESTED WEIGHTINGS AND BLUEPRINTS

These plans and blueprints provide general guidelines for the overall assessment of Pure Mathematics 10.

	Suggested Weightings by Topic		
Topic		**Specific Outcomes (Summary)**	**Suggested Per Cent Weighting**
Sequences and Data Tables	1.1	Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively.	15–17%
	1.2	Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively.	
	1.3	Choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population.	
	1.4	Defend or oppose inferences and generalizations about populations, based on data from samples.	
	1.5	Create and modify tables from both recursive and nonrecursive situations.	
	1.6	Use and modify a spreadsheet template to model recursive situations.	
	1.7	Generate number patterns exhibiting arithmetic growth.	
	1.8	Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems.	
	1.9	Generate number patterns exhibiting geometric growth.	
Algebraic Expressions	2.1	Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$.	24–28%
	2.2	Find the product of polynomials.	
	2.3	Divide an integral polynomial by a binomial.	
	2.4	Determine equivalent forms of single-variable rational expressions with polynomial numerators and denominators.	
	2.5	Determine the nonpermissible values for the variable in single-variable rational expressions.	
	2.6	Perform the operations of addition, subtraction, multiplication and division on single-variable rational expressions.	
	2.7	Find and verify the solutions of rational equations that reduce to linear equations.	
Line Segments and Graphs	3.1	Plot linear and nonlinear data, using appropriate scales.	12–15%
	3.2	Solve problems involving distances between points in the coordinate plane.	
	3.3	Solve problems involving midpoints of line segments.	
	3.4	Solve problems involving rise, run and slope of line segments.	
	3.5	Determine the equation of a line, given information that uniquely determines the line.	
	3.6	Solve problems using slopes of parallel lines and perpendicular lines.	
Relations and Functions	4.1	Represent data, using function models.	15–17%
	4.2	Use a graphing tool to draw the graph of a function from its equation.	
	4.3	Describe a function.	
	4.4	Use function notation to evaluate and represent functions.	
	4.5	Determine the domain and range of a relation from its graph.	
	4.6	Determine characteristics of the graph of a linear function, given its equation.	
	4.7	Use direct variation, partial variation, and arithmetic sequences as applications of linear functions.	
	4.8	Relate arithmetic sequences to linear functions defined over the natural numbers.	
Exponents and Radicals	5.1	Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system.	12–15%
	5.2	Use approximate representations of irrational numbers.	
	5.3	Communicate a set of instructions used to solve an arithmetic problem.	
	5.4	Perform arithmetic operations on irrational numbers, using appropriate decimal approximations.	
	5.5	Explain and apply the exponent laws for powers of numbers and for variables with rational exponents.	
	5.6	Perform operations on irrational numbers of monomial and binomial form, using exact values.	
Trigonometry	6.1	Solve problems involving two right triangles.	13–15%
	6.2	Extend the concepts of sine and cosine for angles from 0° to 180°.	
	6.3	Apply the sine and cosine laws, excluding the ambiguous case, to solve problems.	

Suggested Per Cent Weightings by Standard and Skill				
	Conceptual Understanding	**Procedural Knowledge**	**Problem-solving Skills**	**Totals**
Acceptable Standard	≅20%	≅30%	≅25%	75–80%
Standard of Excellence	≅10%	≅5%	≅10%	20–25%
Totals	25–30%	35–40%	30–35%	100%

TOPICS RELATED TO BASIC RESOURCES

Topic	**Addison-Wesley *Mathematics 10* (Chapter)**	**McGraw-Hill Ryerson *Mathpower 10* (Chapter)**	**Distance Learning Pure Mathematics 10 (Module)**	***The Learning Equation 10* (Lessons)**
Sequences and Data Tables	1 and 9	2 and 8	4, 1 (Section 1 only)	1, 2, 22–24, 44, 45
Algebraic Expressions	6 and 7	3 and 4	3	25–33
Line Segments and Graphs	3 and 4	6	2	17–21
Relations and Functions	5	5	2	34–43
Exponents and Radicals	2	1	4	3–10
Trigonometry	6 and 8	7	5 (Section 2 only)	13–16

TECHNOLOGY REQUIREMENTS

To meet the outcomes of Pure Mathematics 10, students will benefit if they have access to a graphing calculator and a computer with a spreadsheet program. In most classrooms, students will not use technology on a daily basis; however, there are some topics of study for which access to technology is recommended. Students will find the transition to Pure Mathematics 20 much easier if they are familiar with the basic features of the graphing calculator. The following table contains descriptions of the technology-related knowledge and skills expected of students in each of the topic areas, with suggested time requirements.

Topic	**Estimated Time Requirement for Access to Technology**	**Technology-related Knowledge and Skills**
Sequences and Data Tables	1–2 hours	***Students will be expected to:*** • use a computer spreadsheet program, including the application of user-defined functions
Algebraic Expressions	none	**Note**: For assessing this topic, a graphing calculator with symbol manipulation capabilities cannot be available to students.
Line Segments and Graphs	4–5 hours	• graph an equation, using a graphing calculator – enter the equation in the $y =$ form – plot the graph
Relations and Functions	4–5 hours	• graph an equation, using a graphing calculator – enter the equation in the $y =$ form – plot the graph • display a table or use the trace function to estimate the domain and range
Exponents and Radicals	1–2 hours	• convert a decimal to a fraction, using a graphing calculator (optional) • use a scientific or graphing calculator to find the root of a number • communicate a set of instructions for using technology to solve a problem
Trigonometry	2–3 hours	• graph a sine or cosine equation, using a graphing calculator – enter the equation in the $y =$ form – plot the graph – determine the domain and range

MATHEMATICAL CONVENTIONS USED IN THE DESCRIPTION OF SAMPLE TASKS

Approximations, Estimates and Exact Values

An *approximation* to an exact value is a decimal representation, correct to several decimal places, used in a problem. Usually, these approximations result from calculator usage. For example, $\sqrt{3}$ may be replaced by the approximation 1.7321… during the computational phase of a problem.

An *estimate* of an exact value is an approximation that can be derived mentally, and used as part of a mental calculation. For example, the conjecture $\sqrt{2}+\sqrt{8}=\sqrt{10}$ can be shown to be false by using the estimates 1.5, 2.8 and 3.1 for $\sqrt{2}, \sqrt{8}$ and $\sqrt{10}$.

The *exact value* of a rational number is of the form $\frac{p}{q}$, where p and q are integers. Unless the decimal representation terminates, the decimal representation will be an approximation. So $-\frac{5}{3}$ is exact, while $-1.666\ldots$ is an approximation.

The *exact value* of an irrational number is expressed in such forms as sin 35°, $\sqrt{3}$ or $\frac{\pi}{6}$. The decimal representation will always be an approximation. So sin 35° is exact, while 0.573576… is an approximation.

Calculator Use in Multipart Problems

The general rule is to keep the calculator running from beginning to end, using all the available decimal places. All answers, both intermediate and final, are reported to the appropriate number of significant figures. However, rounded intermediate answers are **not** to be used as inputs into subsequent calculations. For example, a right-angled triangle with a measured hypotenuse of 11.9 cm and a shortest side of 6.1 cm would have the third side reported as 10.2 cm. However, if the length of the third side were used in the second part of a calculation, the more precise value 10.21763182… would be used as the input value.

Measured and Exact Input Data

If the *input data is measured*, all output data should be recorded in decimal form to the same quality as the least reliable part of the input data. For example, if the input data had 57.235 g for the mass and 17 mL for the volume, the density would be recorded as 3.4 g/mL with two significant digits. As a matter of convention, trigonometric problems with lengths to three significant digits yield angles to one-tenth of a degree, or one-hundredth of a radian.

If the *input data is exact,* all output data should be recorded in fractional form if at all possible. For example, the slope of the line joining the points (6, 11) and (17, 32) should be recorded as $\frac{21}{11}$, which is exact, rather than the decimal form 1.9090….

For Pure Mathematics 10, all trigonometric data can be assumed to be measured, together with most of the estimated probabilities used in calculations of expectation values. All input data in coordinate geometry, algebraic expressions, relations and functions can be assumed to be exact. Most of the input data in number tables and number patterns can be assumed to be exact, except that money outputs are rounded to the nearest dollar or the nearest cent, depending on the context.

If there is any doubt whether any item of input data is exact or measured, assume it to be exact if it is part of a Pure Mathematics 10 problem.

STANDARDS LINKED TO OUTCOMES

The remainder of this document consists of several components that, taken together, provide guidance to teachers about the scope, depth and breadth of Pure Mathematics 10.

Topics

Topics are the mathematical themes that provide a convenient way for organizing concepts. The general and specific outcomes and the associated tasks in this part of the document are grouped according to topics.

General and Specific Outcomes

The general and specific outcomes for Pure Mathematics 10 identify what students are expected to know and be able to do upon completion of the course.

Strands

Strands are the underlying themes running throughout all of the mathematics programs. The Pure Mathematics 10 course includes:

- Number (Number Concepts)
- Number (Number Operations)
- Patterns and Relations (Patterns)
- Patterns and Relations (Variables and Equations)
- Patterns and Relations (Relations and Functions)
- Shape and Space (Measurement)
- Shape and Space (3-D Objects and 2-D Shapes)
- Statistics and Probability (Data Analysis)

Mathematical Processes

These are the critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and to encourage lifelong learning in mathematics. One or more of the following mathematical processes is emphasized for each specific outcome.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

More information on the general and specific outcomes, strands, and mathematical processes can be found in the *Mathematics Applied and Pure Programs Program of Studies.*

Notes

Each specific outcome is followed by a note that provides additional information about the intended depth and breadth for that outcome. Teachers can refer to these for guidance when planning classroom activities and assessments that address the specific outcome at an appropriate level for the course.

Sample Tasks

Linked to each specific outcome are sample tasks. These consist of questions, problems or activities for classroom use with students. Teachers may use these directly or as models for developing their own tasks for group or independent work in the classroom, or as part of formative and summative assessment of students. The sample tasks and suggested solutions were developed and validated by classroom teachers of Pure Mathematics 10, but **they have not been validated with students**. They represent models of the kinds of questions and problems students should be able to solve, and the kinds of activities students should be able to perform to meet the specific outcome to which they are linked.

The following two examples illustrate the components expected in a student's written response to a problem, when a graphing calculator has been used to assist in determining the answer.

Example 1: When asked to solve a problem that asks for the coordinates of key points, from a description of a situation that can be expressed as a function, the following should be included in the student's response:

- the function in $y =$ form
- a sketch of the graph with the scales labelled
- the key points identified on the sketch
- a statement describing the relationship between the calculator screen output and the answer to the problem, within its context.

Example 2: When solving a problem that involves using the regression menu on a graphing calculator to determine the line of best fit for a data set, students can be expected to include the following in the written response:

- the data from input lists
- the regression formula used
- the equation of the line of best fit
- a sketch of the graph with the scales labelled
- a statement describing the relationship between the calculator screen output and the answer to the problem, within its context.

Descriptions of Student Performance (Related to Specific Outcome)

Each specific outcome is followed by a description of student performance at the acceptable standard and at the standard of excellence. These descriptions represent the judgements of practising classroom teachers in regard to how their students would perform on the particular specific outcome.

Descriptions of Student Performance (Related to Sample Tasks)

The sample tasks for a particular specific outcome are followed by a description of student performance at the acceptable standard and at the standard of excellence. These descriptions, though **not based on actual student responses**, represent the judgements of practising classroom teachers in regard to how their students would perform, given these particular tasks.

STANDARDS IN SEQUENCES AND DATA TABLES

GENERAL OUTCOMES

- Analyze the numerical data in a table for trends, patterns and interrelationships.
- Implement and analyze sampling procedures, and draw appropriate inferences from the data collected.
- Describe and apply arithmetic operations on tables to solve problems, using technology as required.
- Generate and analyze number patterns.

SPECIFIC OUTCOMES

1.1 Use words and algebraic expressions to describe the data and the interrelationships in a given table with rows that are not related recursively (not calculated from previous data). [C, CN]

1.2 Use words and algebraic expressions to describe the data and the interrelationships in a given table with rows that are related recursively (calculated from previous data). [C, CN]

1.3 Choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population. [C, PS, R]

1.4 Defend or oppose inferences and generalizations about populations, based on data from samples. [C, PS, R]

1.5 Create and modify tables from both recursive and nonrecursive situations. [PS, T, V]

1.6 Use and modify a spreadsheet template to model recursive situations. [PS, T, V]

1.7 Generate number patterns exhibiting arithmetic growth. [E, R]

1.8 Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems. [CN, PS, R, T]

1.9 Generate number patterns exhibiting geometric growth. [E, R]

Strand: Number (Number Concepts)

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Analyze the numerical data in a table for trends, patterns and interrelationships.

Specific Outcome

1.1 Use words and algebraic expressions to describe the data and the interrelationships in a given table with rows that are not related recursively (not calculated from previous data). [C, CN]

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 2.1.
- Algebraic expressions can include expressions that use variables and expressions that describe formulas from spreadsheets.
- Technology in the form of spreadsheets may be used to address this specific outcome, but the use of technology is not mandatory.
- Teachers may need to address the terms commonly used as headers in tables.
- Units, except for $, should be placed in column headers and not in table entries.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• obtain specific information from a row/column of a table • modify an algebraic expression for the relationship in a table.	• generalize and make predictions from given data in table form • develop an algebraic expression for the relationship in a table.

General and Specific Outcomes

General Outcome

Analyze the numerical data in a table for trends, patterns and interrelationships.

Specific Outcome

1.1 Use words and algebraic expressions to describe the data and the interrelationships in a given table with rows that are not related recursively (not calculated from previous data). [C, CN]

Sample Task

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. Magazine publishers are examining the sales for the month of September. Below is the table with a record of the results.

Magazine Title	Sales	Price	Revenue
Teen Matters	127	$3.95	$501.65
YZ	132	$3.25	$429.00
Big Trucks	98	$5.25	$514.50
Fitness	158	$5.95	$940.10
Get Real	276	$4.65	$1283.40

a. How many magazines were sold in September?
b. What was the total revenue for all magazines?
c. Describe how to calculate the revenue for each magazine, in words and as a formula.
d. The publishers are changing the price of *YZ* to $3.75. What is the projected revenue of *YZ* magazine for the month of October, if sales remain the same?
e. Do you think that *YZ* sales will increase or decrease after the price increase? What effect will this have on *YZ* revenue? Support your answer with calculations.

Solution:

1. a. The total number of magazines sold was 791.
b. The total revenue was $3668.65.
c. Multiply the number of magazines purchased by the price per magazine.
Formula: $R = S \times P$
d. The revenue for *YZ* will be 132 × $3.75 or $495.00 in October, assuming no change in sales.
e. Answers will vary. Students should be expected to justify both their prediction and their calculations.

Descriptions of Student Performance (Related to Sample Task)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• calculate the total number of magazines sold, using addition • calculate the total revenue, using addition • describe how to calculate revenue in words, and devise an appropriate formula • apply the formula to determine the revenue in part d • provide an unsupported answer to part e.	• make predictions for part e, and support the predictions with calculations.

General and Specific Outcomes

General Outcome

Analyze the numerical data in a table for trends, patterns and interrelationships.

Specific Outcome

1.2 Use words and algebraic expressions to describe the data and the interrelationships in a given table with rows that are related recursively (calculated from previous data). [C, CN]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 2.2.
- Algebraic expressions can include expressions that use variables and expressions that describe formulas from spreadsheets.
- Technology in the form of spreadsheets could be used in addressing this specific outcome, but the use of technology is not required. Students are not expected to create spreadsheets for this outcome.
- Although questions involving compound interest are a good application of this specific outcome, questions should be restricted to those involving interest compounded annually.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• describe the relationship between rows in a table, and devise an appropriate formula for this relationship.	• generalize and make predictions from given data in a table.

General and Specific Outcomes

General Outcome

Analyze the numerical data in a table for trends, patterns and interrelationships.

Specific Outcome

1.2 Use words and algebraic expressions to describe the data and the interrelationships in a given table with rows that are related recursively (calculated from previous data). [C, CN]

Sample Task

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

Question:

1. The following table contains information about a loan.

Year	Opening Balance	Interest Rate %	Interest Charged	Annual Payment	Closing Balance
1	$20 000.00	5	$1000.00	$4619.50	$16 380.50
2	$16 380.50	5	$819.03	$4619.50	$12 580.03
3	$12 580.03	5	$629.00	$4619.50	$8 589.53
4	$8 589.53	5	$429.48	$4619.50	$4 399.51
5	$4 399.51	5	$219.98	$4619.50	$0.00

a. How long is the term of the loan?
b. What is the total interest for the term?
c. Describe how the bank calculated the amounts in the last three columns for Year 1 of the table.

Solution:

1. a. The term is 5 years.
b. The total interest is $3097.49.
c. Interest charged = opening balance × 0.05

$$\text{Annual payment} = \frac{\text{amount borrowed} + \text{total interest}}{5}$$

Closing balance = opening balance + interest charged – annual payment

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• interpret the table to correctly determine the term of the loan and the total interest • provide descriptions of how to determine the interest charged and the closing balance for Year 1 of the table.	• provide a complete description of how all values for the last three columns in Year 1 were determined.

General and Specific Outcomes

General Outcome

Implement and analyze sampling procedures, and draw appropriate inferences from the data collected.

Specific Outcome

1.3 Choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population. [C, PS, R]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome refers primarily to sampling techniques, so the formulation of survey questions is only a minor part of this specific outcome.
- The following sampling techniques should be covered: simple random sampling, systematic sampling, stratified random sampling, clustered sampling, convenience sampling and sampling of volunteers (self-selected sample).
- The concept of bias should be understood in an informal manner.
- Precise statistical definitions of *bias*, *reliability* and *validity* are not required.
- Box plots are beyond the scope of this specific outcome.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• define sample and population • decide if a particular sampling procedure is likely to give an unbiased sample • produce survey questions of first-draft quality • identify problems/weaknesses in a survey.	• justify the use of a particular sampling technique • improve sampling techniques to remove bias from a sample • edit survey questions to improve reliability • provide potential solutions/remedies to address problems/weaknesses in a survey.

General and Specific Outcomes

General Outcome

Implement and analyze sampling procedures, and draw appropriate inferences from the data collected.

Specific Outcome

1.3 Choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population. [C, PS, R]

Sample Task

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. An agency is commissioned to find out how students get to a high school, in order to determine the required parking lot expansion. Describe and justify the sampling techniques that would give the most accurate results.
   a. Describe the population and the sample that is to be used.
   b. Describe in detail how an unbiased sample can be selected from the population.
   c. Justify the sampling procedure used.
   d. Write an appropriate survey question or set of questions.

Solution:

1. Answers will vary, but the sampling technique should:
   - address equal proportions of students from each grade—since fewer Grade 10 students drive than Grade 12 students, the proportions must be exact
   - refer to driving and parking.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• describe both the sample and the population • show that a random sample must be used • describe a stratified random sample in a limited way • write a question that may be incomplete or oversimplified.	• justify, using examples, why a stratified random sample is necessary • include subtle points, such as precise wording and the time of year, in the survey questions.

General and Specific Outcomes

General Outcome

Implement and analyze sampling procedures, and draw appropriate inferences from the data collected.

Specific Outcome

1.4 Defend or oppose inferences and generalizations about populations, based on data from samples. [C, PS, R]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

Notes:

- Students should generate their own data. Guidance should be given so that students can quickly gather rich data that allow for many different inferences and generalizations.
- For this specific outcome, an inference is considered as a prediction of a characteristic of the population.
- *Predictions* may include trends and patterns.
- A *generalization* is considered as a summary statement about the population as a whole.
- Generalizations may include elements of prediction, as well as elements of summary.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• make population inferences that relate closely to the sample data • make generalizations that involve summary • summarize trends and patterns.	• express the limitations of an inference • make generalizations that involve prediction • predict trends and patterns.

General and Specific Outcomes

General Outcome

Implement and analyze sampling procedures, and draw appropriate inferences from the data collected.

Specific Outcome

1.4 Defend or oppose inferences and generalizations about populations, based on data from samples. [C, PS, R]

Sample Task

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. A survey question given to students in a variety of high schools in the Northwest Territories asked them how they get to school. The following data were collected.

Method of Getting to School	Percentage of Students (%)
Walk	26.3
Bike	15.0
Drive	18.7
Bus	37.3
Snowmobile/ATV	2.7
Total	100

Agree or disagree with the following inferences. Explain why you agree or disagree with the inference.

a. Students no longer walk to school.
b. Every high school should have snowmobile/all-terrain vehicle (ATV) parking lots.
c. Most high school parking lots are empty, because only 19% of students drive.
d. The data were taken in summer.

Solution:

1. a. This inference is false, as the survey data show 26.3% of students walk to school.
b. This inference is unreasonable, as the number of students going by snowmobile/ATV is low (2.7%); and, in any case, snowmobiles and ATVs can use a car parking lot.
c. The size of the lot is rarely enough for all students to be able to drive and park. Most schools have space for 20% or so of the students, and so the parking lots will be fairly full.
d. This inference is likely to be true, as few students would walk or bike to school in a Northwest Territories' winter.

Descriptions of Student Performance (Related to Sample Task)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• compare a direct inference to the data, such as in part a • interpret data that is easily related to an inference, such as in part b or part d • provide explanations that may be imprecise.	• assess an inference, such as the one in part c, where additional information is required to be brought to the decision • provide explanations that are precise.

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

1.5 Create and modify tables from both recursive and nonrecursive situations. [PS, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 2.7.
- Table formats can be given to students, and modifications can be made by editing table entries.
- At the standard of excellence, table formats are not given.
- Students can be expected to insert/delete rows and columns as part of the modifications.
- Tables used in this specific outcome should be relatively simple and limited to approximately 24 cells.
- The ability of students to work with tables and spreadsheets may depend on their background from junior high school and from career and technology studies courses.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• modify table formats that are given • insert rows and columns to accommodate changed contexts • calculate the amounts for most of the cells in a recursive table.	• devise a table format for a problem • create and modify recursive and nonrecursive tables • complete calculations for recursive tables.

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

1.5 Create and modify tables from both recursive and nonrecursive situations. [PS, T, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. A 15-year-old student wants to save $2000.00 to buy a car when he turns 18. He is able to save $50 a month from his part-time job. In the first year the interest on the investment account is 8% of the balance at the end of the year. For the second year it is 7%, and for the third year it is 8.5%.
   a. Create a table to determine if the student will have enough money to buy a $2000.00 car after three years. The headings for the columns in your table should be:

Column	**Heading**
A	Year
B	Starting Balance
C	Yearly Savings
D	Starting Balance + Yearly Savings
E	Interest Rate (%)
F	Amount of Interest
G	Ending Balance

   b. Explain how to calculate the amounts for the cells in columns **D**, **F** and **G**.

Solution:

1. a.

A	B	C	D	E	F	G
Year	Starting Balance	Yearly Savings	Starting Balance + Yearly Savings	Interest Rate (%)	Amount of Interest	Ending Balance
1	$0.00	$600.00	$600.00	8	$48.00	$648.00
2	$648.00	$600.00	$1248.00	7	$87.36	$1335.36
3	$1335.36	$600.00	$1935.36	8.5	$164.51	$2099.87

The student can afford the car in three years.

b. Column D: take the sum of columns B and C for each row.
Column F: multiply the value in column D by the interest rate, expressed as a decimal, from column E for each row.
Column G: take the sum of columns D and F for each row.

(continued)

General and Specific Outcomes
General Outcome Describe and apply arithmetic operations on tables to solve problems, using technology as required. **Specific Outcome** 1.5 Create and modify tables from both recursive and nonrecursive situations. [PS, T, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Question:

2. Use the table to answer the questions.

Year	Opening Balance	Interest Rate (%)	Interest Earned	Closing Balance
1	$2500.00	6	$150.00	$2650.00
2	$2650.00	6	$159.00	$2809.00
3	$2809.00	6	$168.54	$2977.54
4	$2977.54	6	$178.65	$3156.19

a. Calculate the total interest earned during the four years.
b. Suppose the interest rate increased to 9% in Year 4. What would the closing balance be at the end of that year?
c. Let A dollars represent the initial investment. Write expressions for the interest earned and the closing balance in Year 1.

Solution:

2. a. Total interest earned is either $(150 + 159 + 168.54 + 178.65) or $(3156.19 – 2500). In either case, interest earned is $656.19.
b. The closing balance would be $3245.52.
c. $IE = 0.06A$ for interest earned.
$CB = 1.06A$ or $CB = A + 0.06A$ for closing balance.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• create the table in question 1, part a, with guidance • explain how to calculate the amounts for most of the cells in question 1, part b • answer question 2, parts a and b.	• create the table in question 1, part a, independently • explain how to calculate correct amounts for all of the cells in question 1, part b • correctly answer question 2, part c.

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

1.6 Use and modify a spreadsheet template to model recursive situations. [PS, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 2.8.
- Spreadsheet formats are generally given to students, and modifications are made by editing table entries and formulas.
- For the more difficult problems, spreadsheet formats are not given, and students can be expected to insert/delete rows and columns as part of the modifications.
- The ability of students to work with spreadsheets may depend on their background from junior high school and from career and technology studies courses.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• use spreadsheet formulas to calculate correct amounts for the cells • modify existing formulas in a given spreadsheet template • insert or delete rows and columns as needed.	• modify user-defined formulas in spreadsheets to reflect changes • state or create formulas in modifying spreadsheets.

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

1.6 Use and modify a spreadsheet template to model recursive situations. [PS, T, V]

Sample Task

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. A couple is taking out a loan for some home renovations. The principal amount of the loan is $20 000, and they are given an interest rate of 5%. The loan is for five years, and the bank informs the couple that their annual payments will be $4619.50 to pay off the loan. The bank gave the couple the following table.

	A	B	C	D	E	F
1	Year	Opening Balance	Interest Rate %	Interest Charged	Annual Payment	Closing Balance
2	1	$20 000.00	5	$1000.00	$4619.50	$16 380.50
3	2	$16 380.50	5	$819.03	$4619.50	$12 580.03
4	3	$12 580.03	5	$629.00	$4619.50	$8 589.53
5	4	$8 589.53	5	$429.48	$4619.50	$4 399.51
6	5	$4 399.51	5	$219.98	$4619.50	$0.00

a. The couple budgeted to make annual payments of $3000 and planned to make a lump sum payment for the remainder of the loan at the end of five years. Modify the template provided to calculate the lump sum payment and the total interest paid in the five years.

b. The couple find they cannot afford to make the lump sum payment after five years. How long will it take to pay off the original loan, using $3000 for an annual payment?

c. Unexpectedly, the couple receives gifts and bonuses totalling $1000 in years 3 and 5. They decide to apply this against their loan. Modify the table in part b, and determine how much sooner they will pay off the loan.

Solution:

1. a.

	A	B	C	D	E	F
1	Year	Opening Balance	Interest Rate %	Interest Charged	Annual Payment	Closing Balance
2	1	$20 000.00	5	$1000.00	$3000.00	$18 000.00
3	2	$18 000.00	5	$900.00	$3000.00	$15 900.00
4	3	$15 900.00	5	$795.00	$3000.00	$13 695.00
5	4	$13 695.00	5	$684.75	$3000.00	$11 379.75
6	5	$11 379.75	5	$568.99	$3000.00	$8 948.74

The lump sum payment at the end of five years is $8948.74, and the total interest is $3948.74.

(continued)

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

1.6 Use and modify a spreadsheet template to model recursive situations. [PS, T, V]

Sample Task

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:

1. b.

	A	B	C	D	E	F
1	Year	Opening Balance	Interest Rate %	Interest Charged	Annual Payment	Closing Balance
2	1	$20 000.00	5	$1000.00	$3000.00	$18 000.00
3	2	$18 000.00	5	$900.00	$3000.00	$15 900.00
4	3	$15 900.00	5	$795.00	$3000.00	$13 695.00
5	4	$13 695.00	5	$684.75	$3000.00	$11 379.75
6	5	$11 379.75	5	$568.99	$3000.00	$8 948.74
7	6	$8 948.74	5	$447.44	$3000.00	$6 396.17
8	7	$6 396.17	5	$319.81	$3000.00	$3 715.98
9	8	$3 715.98	5	$185.80	$3000.00	$901.78
10	9	$901.78	5	$45.09	$946.87	$0.00

It will take the couple nine years to pay off the loan, using payments of $3000 per year. In the ninth year the payment is only $946.87.

1. c.

	A	B	C	D	E	F
1	Year	Opening Balance	Interest Rate (%)	Interest Charged	Annual Payment	Closing Balance
2	1	$20 000.00	5	$1000.00	$3000.00	$18 000.00
3	2	$18 000.00	5	$900.00	$3000.00	$15 900.00
4	3	$15 900.00	5	$795.00	$4000.00	$12 695.00
5	4	$12 695.00	5	$634.75	$3000.00	$10 329.75
6	5	$10 329.75	5	$516.49	$4000.00	$6 846.24
7	6	$6 846.24	5	$342.31	$3000.00	$4 188.55
8	7	$4 188.55	5	$209.43	$3000.00	$1 397.98
9	8	$1 397.98	5	$69.90	$1467.88	$0.00
10	9	$–	5	$–	$–	$–

The loan would be paid off 1 year sooner.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• edit the table for the $3000.00 annual payment • interpret the closing balance as the required lump sum payment in question 1, part a • find the total interest paid over the five years in question 1, part a • extend the table, and identify when the closing balance is zero in question 1, part b.	• make any modifications required for question 1, part c.

General and Specific Outcomes

General Outcome

Generate and analyze number patterns.

Specific Outcome

1.7 Generate number patterns exhibiting arithmetic growth. [E, R]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- No formulas are needed for this specific outcome.
- Other sequences may be introduced to assist in clarifying the definition of an arithmetic sequence.
- Students who are familiar with linear functions and the concept of slope, either through work with line segments and straight line graphs or with relations and functions, may have an advantage in this specific outcome. These students should be able to relate the slope of a linear function with the common difference of an arithmetic sequence.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• generate the next three terms, given four consecutive terms of an arithmetic sequence • generate the next three terms of an arithmetic sequence when a simple problem is given in a straightforward manner.	• generate terms for more complex problems.

General and Specific Outcomes

General Outcome

Generate and analyze number patterns.

Specific Outcome

1.7 Generate number patterns exhibiting arithmetic growth. [E, R]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. Find the next three terms for the arithmetic sequence 16, 23, 30, 37, …

Solution:

1. The pattern is determined by adding 7. The next three terms are: 44, 51, 58

Question:

2. A salesperson receives a base salary of $12 000 per year, plus $100 for every unit sold. What is the salary if 4 units are sold?

Solution:

2. $\$12\,000 + 4(\$100) = \$12\,400$

Question:

3. John receives $3 each week for four weeks; for the next four weeks, he receives $6 per week. For the following four weeks, he receives $9 per week. If the pattern continues, how much will he receive for the 24th week?

Solution:

3. The pattern is: $3 for the first 4 weeks
$6 for the next 4 weeks
$9 for the next 4 weeks
$12 for the next 4 weeks
$15 for the next 4 weeks
$18 for the next 4 weeks
The 24th week occurs in the last set where John receives $18.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• answer questions 1 and 2.	• answer question 3.

General and Specific Outcomes

General Outcome

Generate and analyze number patterns.

Specific Outcome

1.8 Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems. [CN, PS, R, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- The primary emphasis in this specific outcome should be on sequences rather than on series.
- Explore the link of this specific outcome to specific outcomes 4.7 and 4.8 on linear functions.
- Students should be introduced to the table menu of a graphing calculator when generating terms of a sequence/series and in graphing these terms.
- Students are not expected to be able to solve systems of linear equations and are not expected to memorize expressions for either the general term or the series sum.
- Students are expected to solve systems of linear equations in Pure Mathematics 20.
- Series problems should be limited to cases where the first term and the common difference are both known.
- The nonrecursive expression to represent the general term, t_n, of an arithmetic sequence is:
  $t_n = a + (n - 1)d$
  where a = the first term and d = the common difference.
- The recursive expression for an arithmetic sequence is $t_1 = a$, $t_n = t_{n-1} + d$.
- The nonrecursive expression to represent the sum of n terms in an arithmetic sum is:
  $S_n = \frac{n}{2}(a + t_n)$
  or $S_n = \frac{n}{2}[2a + (n - 1)d]$.
- The teacher should introduce students to the recursive definition of arithmetic series, such as $S_1 = a$, $t_n = S_n - S_{(n-1)}$.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve sequence problems where both the first term and the common difference are known • solve sequence problems where either the first term or the common difference is unknown, but not both.	• solve series problems where both the first term and the common difference are known • solve sequence problems where both the first term and the common difference are unknown.

General and Specific Outcomes

General Outcome

Generate and analyze number patterns.

Specific Outcome

1.8 Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems. [CN, PS, R, T]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. David starts work for a company at a salary of $17 000 per year, and he receives a raise of $3000 per year every year.

   a. Use the formula $t_n = a + (n-1)d$ to determine the year when David's salary first reaches $50 000.

   b. Use the formula $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to calculate the total amount David earned in his first five years at the company.

Solution:

1. a.

$$\begin{aligned} 50\,000 &= 17\,000 + (n-1)(3000) \\ 50 &= 17 + 3(n-1) \\ 36 &= 3n \\ n &= 12 \text{ years} \end{aligned}$$

b.

$$\begin{aligned} S_n &= \frac{5}{2}\left[2(17\,000) + (5-1)(3000)\right] \\ &= \$115\,000. \end{aligned}$$

Question:

2. A hockey player had a starting salary of $1.4 million per season and earned $100 000 less per season in a six-season career. Calculate the total earned in his career.

(continued)

General and Specific Outcomes
General Outcome Generate and analyze number patterns. **Specific Outcome** 1.8 Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems. [CN, PS, R, T]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

2. Solution 1:
The series is: $1.4 + 1.3 + 1.2 + 1.1 + 1.0 + 0.9 = 6.9$
Total earned is $6.9 million.

Solution 2:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_6 = \frac{6}{2}[2(1.4) + (6-1)(-0.1)]$$
$$= 6.9$$

Total earned is $6.9 million.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• solve question 1, part a.	• solve question 1, part b • solve question 2.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Generate and analyze number patterns.

Specific Outcome

1.9 Generate number patterns exhibiting geometric growth. [E, R]

Notes:

- Formulas for the general term of a geometric sequence are not part of this specific outcome.
- Geometric growth ($r > 1$) and geometric decay ($0 < r < 1$) are included. Oscillating patterns like 3, –6, 12, –24, … are not included.
- General formulas for geometric sequences and series are found in Pure Mathematics 30.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• generate the sequence for geometric growth ($r > 1$) • identify particular terms for geometric growth patterns.	• interpret the terms of sequences exhibiting geometric growth • generate the sequence for geometric decay ($0 < r < 1$) • interpret the terms of sequences exhibiting geometric decay.

General and Specific Outcomes

General Outcome

Generate and analyze number patterns.

Specific Outcome

1.9 Generate number patterns exhibiting geometric growth. [E, R]

Sample Tasks

☐ Conceptual
☐ Procedural
☑ Problem-solving

Question:

1. A deposit of \$5000, put in a bank, increases by 5% each year. How much is in the bank at the end of nine years? Why is the interest higher in the 9th year, compared to the 2nd?

Solution:

1. Sequence of principals is $\$5000$, $\$5000(1.05)$, $\$5000(1.05)^2$, … up to $\$5000(1.05)^9$. The interest is calculated on the principal in each year. In the 9th year, the principal is greater compared to the principal in the 2nd year. After nine years, there is \$7756.64 in the bank. The interest in the 9th year is 5% of $5000(1.05)^9$ or \$369.36. The interest in the 2nd year is 5% of $5000(1.05)^1$ or \$262.50.

Question:

2. A car originally cost \$24 000 and depreciated by 21% per year, every year. When is it first worth less than \$5000?

Solution:

2.

Year	Worth of Car (\$)
0	24 000
1	× 0.79 = 18 960.00
2	× 0.79 = 14 978.40
3	× 0.79 = 11 832.94
4	× 0.79 = 9348.02
5	× 0.79 = 7384.94
6	× 0.79 = 5834.10
7	× 0.79 = 4608.94

After 7 years it is worth less than \$5000.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• solve question 1.	• solve question 2.

STANDARDS IN ALGEBRAIC EXPRESSIONS

GENERAL OUTCOME

- Generalize operations on polynomials to include rational expressions.

SPECIFIC OUTCOMES

2.1 Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$. [E]

2.2 Find the product of polynomials. [E, R]

2.3 Divide an integral polynomial by a binomial, and express the result in the forms:
- $\frac{P}{D} = Q + \frac{R}{D}$
- $P = DQ + R$
- $P(x) = D(x)Q(x) + R.$

[E, R]

2.4 Determine equivalent forms of single-variable rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored. [PS, R]

2.5 Determine the nonpermissible values for the variable in single-variable rational expressions. [C, CN]

2.6 Perform the operations of addition, subtraction, multiplication and division on single-variable rational expressions. [E, R]

2.7 Find and verify the solutions of rational equations that reduce to linear equations. [CN, PS]

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.1 Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$. [E]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Expressions of the form $x^2 + bx + c$ were factored in Grade 9 Mathematics.
- Students must be able to factor expressions such as $ax^2 + bx + c$, $a \neq 1$ and $a^2x^2 - b^2y^2$.
- Included are expressions such as $af^2 + bf + c$, where f is itself a monomial like x^2
- Two variable polynomials, such as $ax^2 + bxy + cy^2$, are not part of this specific outcome
- Students should not spend time on quadratics where the product ac has many factors.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• identify monomial common factors • factor expressions like $ax^2 + bx + c \; (a \neq 1)$ and $a^2x^2 - b^2y^2$.	• factor expressions that require the identification of monomial common factors • factor expressions like, $f^2 - g^2$, $a^2f^2 - g^2$ and $af^2 + bf + c$, where f and g are monomials.

General and Specific Outcomes
General Outcome Generalize operations on polynomials to include rational expressions. **Specific Outcome** 2.1 Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$. [E]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Factor completely $6x^2 - x - 15$.

Solution:

1. $(2x + 3)(3x - 5)$

Question:

2. Factor completely $12x^2 - 2x - 30$.

Solution:

2. $2(6x^2 - x - 15) = 2(3x - 5)(2x + 3)$

Question:

3. Factor completely $4x^2 - 16y^2$.

Solution:

3. $4(x^2 - 4y^2)$
$= 4(x + 2y)(x - 2y)$

Question:

4. Factor completely $4x^3 - x$.

Solution:

4. $4x^3 - x$
$= x(4x^2 - 1)$
$= x(2x + 1)(2x - 1)$

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• factor questions 1 and 3 completely.	• factor question 2 completely • factor question 4 completely.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.2 Find the product of polynomials. [E, R]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

Notes:

- The following are the most complex examples of the polynomials being multiplied:
  - products of three binomials; e.g., $(2x + 5)(x - 3)(x + 2)$
  - binomial × trinomial; e.g., $(2x + 1)(x^2 + 3x - 2)$.
- This specific outcome cannot be assessed if students are allowed to use calculators with symbol manipulation capabilities.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• multiply two binomials • multiply a monomial by a trinomial • apply a product of polynomials to determine area.	• multiply three binomials • multiply a binomial by a trinomial • apply a product of polynomials to determine volume.

General and Specific Outcomes
General Outcome Generalize operations on polynomials to include rational expressions. **Specific Outcome** 2.2 Find the product of polynomials. [E, R]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

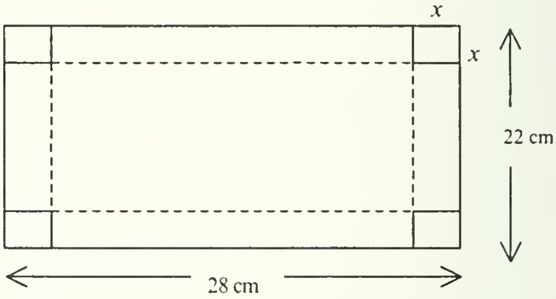
1. A rectangle has sides $(2x + 5)$ cm and $(3x - 2)$ cm.
   a. Find the area of the rectangle.
   b. If the rectangle is a square, what is the length of one side?

Solution:

1. a. Area $= (2x + 5)(3x - 2)$
   $= 6x^2 + 11x - 10$
   b. $2x + 5 = 3x - 2$
   $\therefore x = 7$ and one side is $2(7) + 5$ or 19 cm.

Question:

2. A 22 cm × 28 cm sheet of cardboard has a square of side x cm cut out from each corner. The cardboard is folded along the dotted lines to form a box.



   a. Write monomial or binomial expressions for the length, width and height of the box.
   b. Write an expression in the form $ax^3 + bx^2 + cx + d$ for the volume of the box.

Solution:

2. a. $L = 28 - 2x$ for length.
   $W = 22 - 2x$ for width.
   $H = x$ for height.

   b. $V(x) = x(28 - 2x)(22 - 2x)$
   $= 4x^3 - 100x^2 + 616x$

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete question 1 • complete question 2, part a.	• write the volume in question 2, part b, in the forms: $V(x) = x(28 - 2x)(22 - 2x)$ and $V(x) = 4x^3 - 100x^2 + 616x$.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.3 Divide an integral polynomial by a binomial, and express the result in the forms:

- $\frac{P}{D} = Q + \frac{R}{D}$
- $P = DQ + R$
- $P(x) = D(x)Q(x) + R$.

[E, R]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Divisions are normally limited to the division of any integral polynomial by a binomial of the form $(x - b)$, where b is an integer.
- Also included are divisors of form $(ax - b)$ or $(ax^2 + c)$.
- The Factor Theorem is covered in Pure Mathematics 20.
- Synthetic division is not required for this specific outcome and is optional in Pure Mathematics 20.
- Review of ascending and descending order of polynomials is essential.
- This specific outcome is restricted to integral polynomials for $P(x)$, $D(x)$ and $Q(x)$, with R being either an integer or an integral linear binomial.
- This specific outcome cannot be assessed if students are allowed to use calculators with symbol manipulation capabilities.
- The expressions $P(x)$, $Q(x)$ and $R(x)$ are used to describe the algebraic structures. However, these expressions should not be used with students in this context.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• divide any integral polynomial, where no coefficients are zero, by a binomial of the form $(x - b)$, where b is an integer.	• divide any integral polynomial, with one or more coefficients equal to zero, by a binomial of the form $(x - b)$, where b is an integer • divide any integral polynomial by an integral divisor of the form $(ax - b)$ or $(ax^2 + c)$.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.3 Divide an integral polynomial by a binomial, and express the result in the forms:
- $\frac{P}{D} = Q + \frac{R}{D}$
- $P = DQ + R$
- $P(x) = D(x)Q(x) + R$.

[E, R]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Divide $x^3 + 5x^2 - 2x + 6$ by $x - 2$. Write the result of the division in two different ways.

Solution:

1.
$$x^3 + 5x^2 - 2x + 6 = (x-2)(x^2 + 7x + 12) + 30$$

or

$$\frac{x^3 + 5x^2 - 2x + 6}{x - 2} = x^2 + 7x + 12 + \frac{30}{x - 2}$$

Question:

2. Given a divisor, $D(x) = x^2 + 2$, a quotient, $Q(x) = x - 4$, and a remainder, $R = -3$, what is the original polynomial, $P(x)$?

Solution:

2. $P(x) = (x^2 + 2)(x - 4) + (-3)$
$P(x) = x^3 - 4x^2 + 2x - 11$

Question:

3. The area of a rectangle is $(3x^2 - 10x + 3)$ cm^2, and the width is $(x - 3)$ cm. What is the length of the rectangle?

Solution:

3.
$$L = \frac{A}{W}$$
$$= \frac{(3x^2 - 10x + 3)\text{ cm}^2}{(x-3)\text{ cm}}$$

length = $(3x - 1)$ cm.

(continued)

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.3 Divide an integral polynomial by a binomial, and express the result in the forms:

- $\frac{P}{D} = Q + \frac{R}{D}$
- $P = DQ + R$
- $P(x) = D(x)Q(x) + R$.

[E, R]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

4. Divide $x^3 + 5x^2 + 6$ by $x - 2$. Write the result of the division in two different ways.

Solution:

4.
$$x^3 + 5x^2 + 6 = (x-2)(x^2 + 7x + 14) + 34$$

or

$$\frac{x^3 + 5x^2 + 6}{x - 2} = x^2 + 7x + 14 + \frac{34}{x-2}$$

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete the division in question 1, and express the solution in at least one of the acceptable forms • demonstrate that the solution to question 2 involves multiplying $D(x)$ by $Q(x)$ • solve question 3 completely.	• express the solution to question 1 in both acceptable forms • solve question 2 completely • solve question 4 completely.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.4 Determine equivalent forms of single-variable rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored. [PS, R]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Nonpermissible values are covered under specific outcome 2.5.
- This specific outcome and specific outcome 2.5 can be covered simultaneously.
- The expressions must be of the form $\dfrac{P(x)}{Q(x)}$ or $\dfrac{P(x)Q(x)}{R(x)}$.
- This specific outcome cannot be assessed if students are allowed to use calculators with symbol manipulation capabilities.
- The expressions $P(x)$, $Q(x)$ and $R(x)$ are used to describe the algebraic structures. However, these expressions should not be used with students in this context.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• reduce rational expressions involving denominators that are monomials, a product of binomial factors or a trinomial of the form $x^2 + bx + c$.	• reduce rational expressions involving denominators of the form $ax^2 + bx + c$, where $a \neq 1$ • reduce rational expressions where polynomials of the form $af^2 + bf + c$, with f monomial, are included.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.4 Determine equivalent forms of single-variable rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored. [PS, R]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Simplify the following expressions, using factoring.

Question:	*Solution:*
1. $\dfrac{2x^3-8x}{2x}$	$\dfrac{2x(x^2-4)}{2x}=(x-2)(x+2)$ **or** x^2-4
2. $\dfrac{3x^2+17x+10}{(3x+2)^3}$	$\dfrac{(3x+2)(x+5)}{(3x+2)^3}=\dfrac{(x+5)}{(3x+2)^2}$
3. $\dfrac{3x^2-11x+6}{x^2-7x+12}$	$\dfrac{(3x-2)(x-3)}{(x-4)(x-3)}=\dfrac{3x-2}{x-4}$
4. $\dfrac{x^2-5x+6}{x^4-13x^2+36}$	$\dfrac{(x-3)(x-2)}{(x^2-9)(x^2-4)}$ $=\dfrac{(x-3)(x-2)}{(x-3)(x+3)(x-2)(x+2)}$ $=\dfrac{1}{(x+2)(x+3)}$

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• solve questions 1, 2 and 3.	• solve question 4.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.5 Determine the nonpermissible values for the variable in single-variable rational expressions. [C, CN]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- The expressions can be single terms of the form $\frac{P(x)}{Q(x)}$, or expressions such as: $\frac{P(x)}{Q(x)}+\frac{R(x)}{T(x)}, \frac{P(x)}{Q(x)}\times\frac{R(x)}{T(x)}$ or $\frac{P(x)}{Q(x)}\div\frac{R(x)}{T(x)}$. Factors of $P(x)$, $Q(x)$, $R(x)$ and $T(x)$ should be easily identified.
- Students should be able to find nonpermissible values when the:
  - denominator is a binomial of the form $ax + b$, and $a \neq 1$
  - denominator must be factored first
  - expression is of the form $\frac{a}{b} \div \frac{c}{d}$
  - denominator must be factored first **and** contains a factor of the numerator
  - nonpermissible values are expressed in terms of another variable.
- This specific outcome should be integrated with specific outcomes 2.4 and 2.6.
- The expressions $P(x)$, $Q(x)$, $R(x)$ and $T(x)$ are used to describe the algebraic structures. However, these expressions should not be used with students in this context.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• determine all the nonpermissible values where the denominator is easily factored and the original expression is in its lowest terms and of the form $\frac{P(x)}{Q(x)}$, $\frac{P(x)}{Q(x)}\pm\frac{R(x)}{T(x)}$ or $\frac{P(x)}{Q(x)}\times\frac{R(x)}{T(x)}$ • determine some of the nonpermissible values when $\frac{P(x)}{Q(x)}$ is not in its lowest terms.	• determine all the nonpermissible values when the original expression is not in its lowest terms • determine all the nonpermissible values for such expressions as: $\frac{P(x)}{Q(x)}\div\frac{R(x)}{T(x)}$.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.5 Determine the nonpermissible values for the variable in single-variable rational expressions. [C, CN]

Sample Task

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

Question:

1. Determine the nonpermissible values for x in the following expressions:

   a. denominator is a binomial of the form $ax + b$, and $a \neq 1$
   $$\frac{3}{2x+5}$$

   b. denominator must be factored first
   $$\frac{2x+1}{x^2-2x-3}$$

   c. expression is of the form $\frac{a}{b} \div \frac{c}{d}$
   $$\frac{x^2}{x-4} \div \frac{x^2-9}{x^2-25}$$

   d. denominator must be factored first and contains a factor of the numerator
   $$\frac{2x+1}{2x^2+7x+3}$$

Solution:

1. a. $2x + 5 \neq 0$, so $x \neq -\frac{5}{2}$.

   b. $$\frac{2x+1}{(x-3)(x+1)}$$
   $x - 3 \neq 0$
   $x \neq 3$
   $x + 1 \neq 0$
   $x \neq -1$, so $x \neq -1, 3$

   c. $$\frac{x^2}{x-4} \div \frac{(x-3)(x+3)}{(x-5)(x+5)}$$
   $x - 4 \neq 0$ $\quad$ $x \neq 4$
   $(x-5)(x+5) \neq 0$ $\quad$ $x \neq \pm 5$
   $(x-3)(x+3) \neq 0$ $\quad$ $x \neq \pm 3$
   So the set is $x \neq \pm 3, 4, \pm 5$.

(continued)

General and Specific Outcomes
General Outcome Generalize operations on polynomials to include rational expressions. **Specific Outcome** 2.5 Determine the nonpermissible values for the variable in single-variable rational expressions. [C, CN]

Sample Task

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

(continued)

Solution:

d. $\dfrac{(2x+1)}{(2x+1)(x+3)}$

$2x + 1 \neq 0 \qquad x \neq -\dfrac{1}{2}$

$x + 3 \neq 0 \qquad x \neq -3$

so $x \neq -3, \ -\dfrac{1}{2}$

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• determine the complete set of nonpermissible values in question 1, parts a and b • determine $x \neq 4$ and $x \neq \pm 5$, omitting $x \neq \pm 3$ in question 1, part c • determine $x \neq -3$, by finding the nonpermissible values after reduction of the fraction in question 1, part d.	• determine the complete set of nonpermissible values in question 1, parts c and d.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.6 Perform the operations of addition, subtraction, multiplication and division on single-variable rational expressions. [E, R]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

Notes:

- Relevant discussion of nonpermissible values is expected as part of this specific outcome.
- Division problems must be expressed in the form $\frac{P(x)}{Q(x)} \div \frac{R(x)}{T(x)}$.
- Complex fractions of the form $\frac{P(x)+\frac{Q(x)}{R(x)}}{T(x)}$, or the equivalent, are not part of this specific outcome.
- Order of operations is not part of this specific outcome.
- The rational expression should be expressed in terms of a single variable.
- This specific outcome cannot be assessed if students are allowed to use calculators with symbol manipulation capabilities.
- The expressions $P(x)$, $Q(x)$, $R(x)$ and $T(x)$ are used to describe the algebraic structures. However, these expressions should not be used with students in this context.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• write $\frac{P(x)}{Q(x)}$ in an equivalent form, by multiplying the numerator and the denominator by a monomial or a binomial • perform calculations on expressions containing factors of the form $ax^2 + bx + c$ or $x + c$ • provide answers in unreduced form • use the common denominator, rather than the lowest common denominator, for expressions of the type $\frac{a}{b} + \frac{c}{d}$, with b and d sharing a common factor.	• reduce answers to lowest terms • use the lowest common denominator for expressions of the type $\frac{a}{b} + \frac{c}{d}$, with b and d sharing a common factor • solve division questions, written in the form $\frac{a}{b} \div \frac{c}{d}$.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.6 Perform the operations of addition, subtraction, multiplication and division on single-variable rational expressions. [E, R]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Perform the indicated operations for each of the following expressions. Express final answers in lowest terms, and indicate the nonpermissible values.

Question:

1. $\frac{1}{x} - \frac{1}{3x}$

Solution:

1. $\frac{1}{x} - \frac{1}{3x}; x \neq 0$

$$= \frac{3}{3x} - \frac{1}{3x}$$

$$= \frac{2}{3x}; x \neq 0$$

Question:

2. $\frac{3}{x+2} + \frac{x}{5}$

Solution:

2. $\frac{3}{x+2} + \frac{x}{5}; x \neq -2$

$$= \frac{(3)(5)}{5(x+2)} + \frac{x(x+2)}{5(x+2)}$$

$$= \frac{x^2 + 2x + 15}{5(x+2)}; x \neq -2$$

(continued)

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.6 Perform the operations of addition, subtraction, multiplication and division on single-variable rational expressions. [E, R]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

3. $\dfrac{4x}{x+2} + \dfrac{3}{x-1}$

Solution:

3. $\dfrac{4x}{x+2} + \dfrac{3}{x-1}$; $x \neq -2, 1$

$$= \frac{4x(x-1)}{(x+2)(x-1)} + \frac{3(x+2)}{(x+2)(x-1)}$$

$$= \frac{4x^2 - 4x}{(x+2)(x-1)} + \frac{3x+6}{(x+2)(x-1)}$$

$$= \frac{4x^2 - x + 6}{(x+2)(x-1)}; x \neq -2, 1$$

Question:

4. $\dfrac{x}{x-3} - \dfrac{x-1}{x+4}$

Solution:

4. $\dfrac{x}{x-3} - \dfrac{x-1}{x+4}$; $x \neq 3, -4$

$$= \frac{x(x+4)}{(x-3)(x+4)} - \frac{(x-1)(x-3)}{(x-3)(x+4)}$$

$$= \frac{(x^2+4x) - (x^2-4x+3)}{(x-3)(x+4)}$$

$$= \frac{8x-3}{(x-3)(x+4)}; x \neq 3, -4$$

(continued)

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.6 Perform the operations of addition, subtraction, multiplication and division on single-variable rational expressions. [E, R]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

5. $\left[\frac{x^2-3x-10}{2x}\right]\left[\frac{x^2-3x}{x-5}\right]$

Solution:

5. $$\left[\frac{x^2-3x-10}{2x}\right]\left[\frac{x^2-3x}{x-5}\right]; x \neq 0, 5$$
$$= \left[\frac{(x-5)(x+2)}{2x}\right]\left[\frac{x(x-3)}{(x-5)}\right]$$
$$= \frac{(x+2)(x-3)}{2}; x \neq 0, 5$$

Question:

6. $\frac{x^2-16}{x^2+7x+12} \div \frac{x^2-2x-8}{2x+4}$

Solution:

6. $$\frac{x^2-16}{x^2+7x+12} \div \frac{x^2-2x-8}{2x+4}; x \neq \pm 4, -3, -2$$
$$= \frac{(x-4)(x+4)}{(x+4)(x+3)} \div \frac{(x-4)(x+2)}{2(x+2)}; x \neq 4, -2, -4, -3$$
$$= \frac{(x-4)(x+4)}{(x+4)(x+3)} \times \frac{2(x+2)}{(x-4)(x+2)}$$
$$= \frac{2}{x+3}; x \neq \pm 4, -3, -2$$

(continued)

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.6 Perform the operations of addition, subtraction, multiplication and division on single-variable rational expressions. [E, R]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

7. $\dfrac{12x}{x^2-9}+\dfrac{2x}{x+3}$

Solution:

7. Solution 1:

$$\frac{12x}{x^2-9}+\frac{2x}{x+3}$$
$$=\frac{12x}{(x-3)(x+3)}+\frac{2x}{(x+3)};\ x\neq\pm3$$
$$=\frac{12x}{(x-3)(x+3)}+\frac{2x(x-3)}{(x-3)(x+3)}$$
$$=\frac{2x^2+6x}{(x-3)(x+3)}$$
$$=\frac{2x(x+3)}{(x-3)(x+3)}$$
$$=\frac{2x}{(x-3)};\ x\neq\pm3$$

Solution 2:

$$\frac{12x}{x^2-9}+\frac{2x}{x+3}$$
$$=\frac{12x(x+3)}{(x^2-9)(x+3)}+\frac{2x(x^2-9)}{(x^2-9)(x+3)};\ x\neq\pm3$$
$$=\frac{12x^2+36x+2x^3-18x}{(x^2-9)(x+3)}$$
$$=\frac{2x^3+12x^2+18x}{(x^2-9)(x+3)};\ x\neq\pm3$$

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• complete the calculations for questions 1, 2, 3 and 4 • leave answers to questions 5, 6 and 7 in unreduced form, such as solution 2 in question 7 • identify all the nonpermissible values in questions 1, 2, 3 and 4.	• produce fully-reduced answers to questions 5 and 6 • complete question 7 to solution 1 • identify all the nonpermissible values in questions 5, 6 and 7.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.7 Find and verify the solutions of rational equations that reduce to linear equations. [CN, PS]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

Notes:

- Students are not expected to solve quadratic equations. As a result, the number of realistic problems for this outcome is very limited.
- Rational equations will be investigated further in Pure Mathematics 20.
- Do not spend too much time on this specific outcome, as this topic is revisited in Pure Mathematics 20.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve for x in rational equations containing denominators that are monomials or of the form $x + c$ • participate in and contribute toward the problem-solving process for problems that require the application of rational equations.	• complete the solution to problems that require the application of rational equations.

General and Specific Outcomes

General Outcome

Generalize operations on polynomials to include rational expressions.

Specific Outcome

2.7 Find and verify the solutions of rational equations that reduce to linear equations. [CN, PS]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. Solve for x.

$$\frac{x}{x+6} - \frac{2}{x} = 1$$

Solution:

1. $x^2 - 2(x+6) = x(x+6)$

$$-2x - 12 = 6x$$

$$x = -\frac{3}{2}$$

Question:

2. A carpet of area 240 m^2 has its length doubled. The width is reduced by 5 m. Calculate the original dimensions of the carpet.

Solution:

2. Let L be the original carpet length.

$$\begin{aligned}\frac{240}{L} - \frac{240}{2L} &= 5\\ 480 - 240 &= 10L\\ L &= 24 \text{ m}\end{aligned}$$

The original carpet is 24 m long by 10 m wide.

Question:

3. A motorist increases the average speed of his car on a 300 km journey by 20% and saves 30 minutes. What is the original speed? Check your solution. Note: Speed $(v) = \dfrac{\text{distance } (d)}{\text{time } (t)}$.

Solution:

3.

	Distance (km)	Speed (km/h)	Time (h)
Old	300	v	t
New	300	$1.20v$	$(t - 0.5)$

$$\frac{300}{v} - \frac{300}{1.20v} = 0.5$$

$300(1.20v) - 300v = (0.5)(1.20v)$

$60v = 0.60v$

$$\frac{60}{0.60} = v$$

Original speed = 100 km/h.

Check: Old time = (300 km)/(100 km/h) = 3.0 h
New time = (300 km)/(120 km/h) = 2.5 h
Difference = 0.5 h

(continued)

General and Specific Outcomes
General Outcome Generalize operations on polynomials to include rational expressions. **Specific Outcome** 2.7 Find and verify the solutions of rational equations that reduce to linear equations. [CN, PS]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Question:

4. Sally scored 56 marks on her first test, and 44 out of 50 on her second test. Both tests were weighted equally, and Sally's average mark was 79%. What was the total number of marks possible for the first test?

Solution:

4. Sally's marks were $\frac{56}{x}$ and $\frac{44}{50}$. The average of these is 79%, or $\frac{79}{100}$.

   So the equation describing this connection is: $\frac{1}{2}\left(\frac{56}{x}+\frac{44}{50}\right)=\frac{79}{100}$.

   Multiply by 2, and use a decimal on the right: $\left(\frac{56}{x}+\frac{44}{50}\right)=1.58$

   Multiply by $50x$ to clear fractions: $(56)(50)+44x=1.58(50x)$
   $2800+44x=79x$
   $35x=2800$, or $x=80$
   So the total number of marks possible for the first test was 80.

   Check:

   First test – 56 out of 80, or 70%; second test – 44 out of 50, or 88%. These average 79%, so the solution checks out.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• solve questions 1 and 2 • calculate the new speed in question 3 • calculate the total number of marks possible for the first test, if the equation is given in question 4.	• complete question 3, including a check of the solution • set up the equation in question 4 • solve question 4, including a check of the solution.

STANDARDS IN LINE SEGMENTS AND GRAPHS

GENERAL OUTCOMES

- Examine the nature of relations with an emphasis on functions.
- Solve coordinate geometry problems involving lines and line segments.

SPECIFIC OUTCOMES

3.1 Plot linear and nonlinear data, using appropriate scales. [C, V]

3.2 Solve problems involving distances between points in the coordinate plane. [PS, V]

3.3 Solve problems involving midpoints of line segments. [PS]

3.4 Solve problems involving rise, run and slope of line segments. [PS, V]

3.5 Determine the equation of a line, given information that uniquely determines the line. [PS, V]

3.6 Solve problems using slopes of:
  - parallel lines
  - perpendicular lines.

[CN, PS, V]

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use patterns to describe the world and to solve problems.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.1 Plot linear and nonlinear data, using appropriate scales.
[C, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 3.1.
- Data collection can be time consuming; it is appropriate to provide data in most instances.
- It is important to discuss whether or not it is appropriate to connect points on a graph.
- An optional extension of this specific outcome might be to use the regression model and the graphing calculator to determine a line of best fit function for the data.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• plot data using paper and pencil as well as a graphing calculator • plot a graph, given a table of values or a description of a relationship • choose appropriate scales, axis labels and titles for graphs.	• choose appropriate window settings when using a graphing calculator to graph • draw inferences from data and a graph of the data.

General and Specific Outcomes
General Outcome Examine the nature of relations with an emphasis on functions. **Specific Outcome** 3.1 Plot linear and nonlinear data, using appropriate scales. [C, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

Question:

1. The diameter (D) of several circular objects was measured, and the circumference (C) was calculated. The results are in the following table.

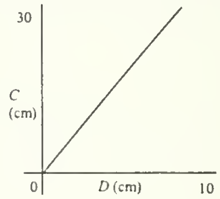
D (cm)	C (cm)
3.0	9.4
5.0	15.7
7.0	22.0
9.0	28.3

a. Graph the data.
b. Is this a linear or nonlinear relationship?
c. Is (0, 0) a point on this graph? Explain why or why not.

Solution:

1. a.

Circumference and Diameter of Circular Objects



b. This is a linear relationship.
c. This graph will have a (0, 0) point; when D is equal to 0, C is 0.
OR
The origin (0, 0) is not a point on this graph, because it is not reasonable to have an object with a diameter of 0.

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.1 Plot linear and nonlinear data, using appropriate scales. [C, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

(continued)
Question:

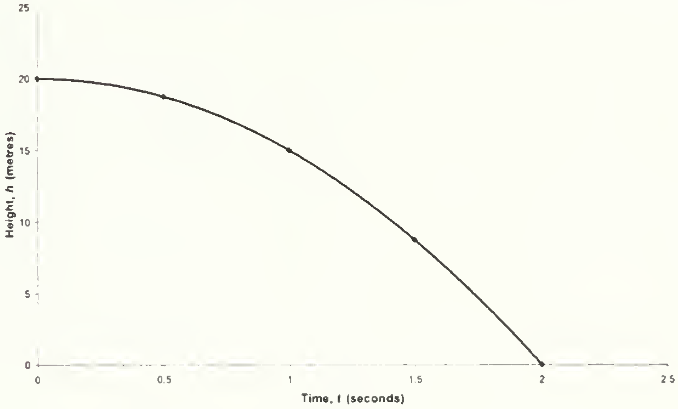
2. The following data was collected for an object dropped from a height above the ground, in metres, over time, t, in seconds.

t (s)	h (m)
0	20.00
0.5	18.75
1.0	15.00
1.5	8.75
2.0	0

a. Graph this data.
b. Is this a linear or nonlinear relationship? Explain your answer in terms of the motion of the object.
c. Explain why t cannot be greater than 2.

Solution:

2. a. **Height as a Function of Time for a Falling Object**



b. This is a nonlinear relationship. The speed of the object is changing.
c. The value of t cannot exceed 2, because the object hits the ground at 2 seconds.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• graph the data for question 1 but may include negative values or treat the data as discrete • graph the data for question 2, using an appropriate scale • recognize question 1 as a linear relationship and question 2 and a nonlinear relationship • provide an explanation for question 2, part c.	• produce a graph with no errors for question 1 • provide an explanation for question 1, part c, and for question 2, part b.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.2 Solve problems involving distances between points in the coordinate plane. [PS, V]

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 4.1.
- It is recommended that teachers review with students the coordinate grid system, ordered pairs and Pythagorean theorem from Grade 9 Mathematics.
- When using the distance formula, students would be expected to show the initial substitution, but it is not necessary to show intermediate steps before the final answer.
- Students are not expected to solve any quadratic equations that may occur through use of the distance formula.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• determine an expression for distance **OR** • plot points on a coordinate plane, and use the Pythagorean theorem to determine distance.	• solve distance problems using a variety of strategies, when intermediate steps are not provided.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.2 Solve problems involving distances between points in the coordinate plane. [PS, V]

Sample Tasks

☐ Conceptual
☑ Procedural
☐ Problem-solving

Question:

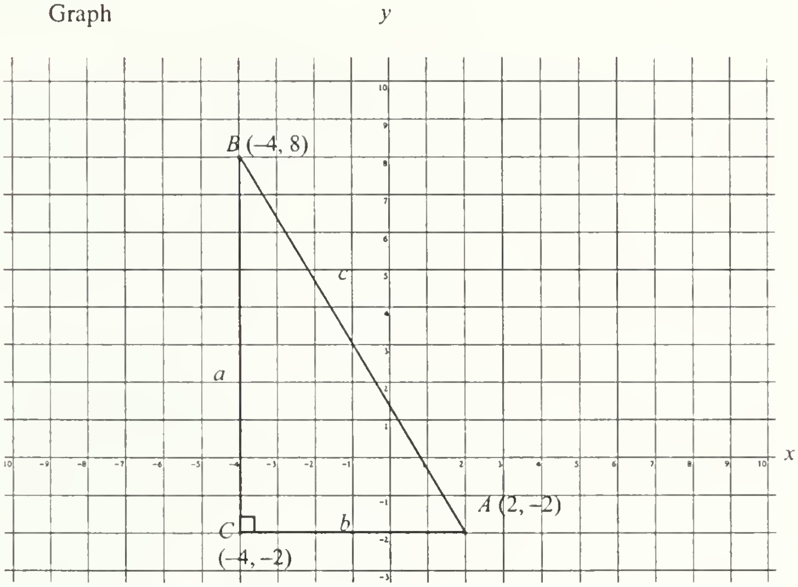
1. Given $A(2, -2)$ and $B(-4, 8)$ on a coordinate grid, determine the distance between A and B. Express the distance as a simplified radical.

Solution:

1.
$$d = \sqrt{(8-(-2))^2 + ((-4)-2)^2}$$
$$d = \sqrt{(10)^2 + (-6)^2}$$
$$d = \sqrt{100+36}$$
$$d = \sqrt{136} \text{ or } 2\sqrt{34}.$$

OR

Graph



$a = 10$
$b = 6$
Then:

$$c = \sqrt{a^2+b^2}$$
$$= \sqrt{10^2+6^2}$$
$$= \sqrt{136}$$
$$= 2\sqrt{34}$$

(continued)

General and Specific Outcomes
General Outcome Solve coordinate geometry problems involving lines and line segments. **Specific Outcome** 3.2 Solve problems involving distances between points in the coordinate plane. [PS, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

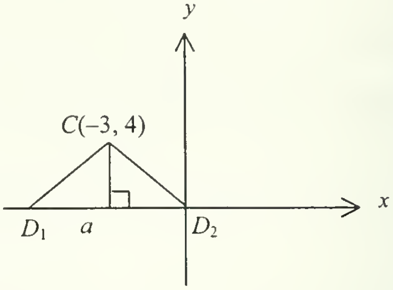
(continued)

Question:

2. Given that the distance between C and D is 5, and point C is at (–3, 4) on a coordinate grid, determine the coordinates of D, if D lies on the x-axis.

Solution:

2.



$$\begin{aligned} a^2 + 4^2 &= CD^2 \\ a^2 + 4^2 &= 5^2 \\ a &= 3 \end{aligned}$$

$D_1 = (-6, 0)$ and $D_2 = (0, 0)$.

Question:

3. A triangular piece of ranch property is listed for sale. It has coordinates of $A(21, 67)$, $B(26, 73)$ and $C(28, 59)$. Assume the coordinates are listed in kilometres.

   a. Determine the perimeter of the property.
   b. A realtor walked around the perimeter of the property at an average speed of 6 km/h. How long would it take the realtor to walk around the perimeter?

   Note: Formula for speed: $v = \frac{\text{distance}}{\text{time}}$.

(continued)

General and Specific Outcomes
General Outcome Solve coordinate geometry problems involving lines and line segments. **Specific Outcome** 3.2 Solve problems involving distances between points in the coordinate plane. [PS, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

3. a. $AB = \sqrt{(26-21)^2 + (73-67)^2}$
$= 7.8$ km
$BC = \sqrt{(28-26)^2 + (59-73)^2}$
$= 14.1$ km
$CA = \sqrt{(21-28)^2 + (67-59)^2}$
$= 10.6$ km
$P = AB + BC + CA$
$= 7.8$ km + 14.1 km + 10.6 km
$= 32.5$ km

b. $t = \frac{d}{v}$
$= (32.5$ km$)/(6$ km/h$)$
$\doteq 5.42$ h or 5 h 25 min.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• determine the distance in question 1, using the distance formula, but expresses it as a decimal or as an entire radical **OR** • plot the points on a coordinate plane, and use the Pythagorean theorem to solve question 1 • determine the sides of the triangle for question 2, by: – plotting *C* – plotting a point *D* on the *x*-axis – setting up a right triangle • determine the perimeter of the property in question 3, part a, if prompts are provided in the form of intermediate steps.	• determine the distance in question 1, and express it as a simplified radical • determine the coordinates of at least one of the possibilities for *D* in question 2; find the second possibility for *D*, given direction • determine the perimeter and time in question 3, without prompts.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.3 Solve problems involving midpoints of line segments. [PS]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 4.2.
- Points on line segments should be described in terms of ordered (coordinate) pairs.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• find the midpoint of a line segment, given the endpoints • find an endpoint of a line segment, given the other endpoint and the midpoint.	• solve multistep problems involving midpoint and other geometrical concepts.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.3 Solve problems involving midpoints of line segments. [PS]

Sample Tasks

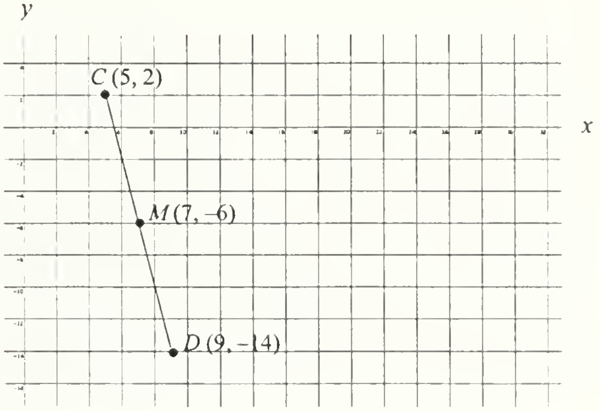
[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. A grain terminal is to be constructed at the midpoint between two rural centres connected by a railway. The coordinates of the centres are (5, 2) and (9, –14). What are the coordinates of the grain terminal?

Solution:

1.



The grain terminal is at (7, –6).

OR

$$M = \left(\frac{5+9}{2}, \frac{(2+(-14))}{2}\right)$$

$= (7, -6)$; these are the coordinates of the grain terminal.

Question:

2. Given $C(5, 2)$ and the midpoint of $\overline{CD}$ at (7, –6), determine the coordinates of D.

Solution:

2. $C(5, 2)$; $M(7, -6)$; $D(x, y)$

$$\begin{aligned} \frac{x+5}{2} &= 7 & \frac{y+2}{2} &= -6 \\ x+5 &= 14 & y+2 &= -12 \\ x &= 9 & y &= -14 \end{aligned}$$

The coordinates of D are (9, –14).

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.3 Solve problems involving midpoints of line segments. [PS]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

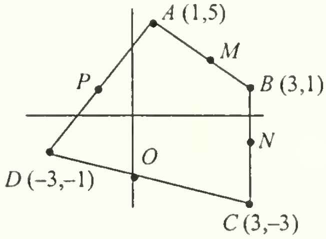
(continued)

Question:

3. A quadrilateral has vertices $A(1, 5)$, $B(3, 1)$, $C(3, -3)$ and $D(-3, -1)$. Determine the perimeter of the quadrilateral formed by joining the **midpoints** of the sides of the quadrilateral $ABCD$. Express the final answer as a simplified radical.

Solution:

3.



(1, 5) and (3, 1)

$$M_1 = \left(\frac{3+1}{2}, \frac{1+5}{2}\right) = (2, 3)\ M$$

(3, 1) and (3, −3)

$$M_2 = \left(\frac{3+3}{2}, \frac{1+(-3)}{2}\right) = (3, -1)\ N$$

(3, −3) and (−3, −1)

$$M_3 = \left(\frac{-3+3}{2}, \frac{-1+(-3)}{2}\right) = (0, -2)$$

O

(−3, −1) and (1, 5)

$$M_4 = \left(\frac{-3+1}{2}, \frac{-1+5}{2}\right) = (-1, 2)\ P$$

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.3 Solve problems involving midpoints of line segments. [PS]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

3. $\overline{MN}$ = (2, 3) and (3, –1)

$$\overline{MN} = \sqrt{(3-2)^2 + (-1-3)^2}$$
$$= \sqrt{1^2 + (-4)^2}$$
$$= \sqrt{1+16} = \sqrt{17}$$

$\overline{OP}$ = (0, –2) and (–1, 2)

$$\overline{OP} = \sqrt{(-1-0)^2 + (2-(-2))^2}$$
$$= \sqrt{(-1)^2 + 4^2}$$
$$= \sqrt{1+16} = \sqrt{17}$$

$\overline{NO}$ = (3, –1) and (0, –2)

$$\overline{NO} = \sqrt{(0-3)^2 + (-2-(-1))^2}$$
$$= \sqrt{(-3)^2 + (-1)^2}$$
$$= \sqrt{9+1} = \sqrt{10}$$

$\overline{MP}$ = (2, 3) and (–1, 2)

$$\overline{MP} = \sqrt{(-1-2)^2 + (2-3)^2}$$
$$= \sqrt{(-3)^2 + (-1)^2}$$
$$= \sqrt{9+1} = \sqrt{10}$$

$$\text{Perimeter} = \sqrt{17} + \sqrt{17} + \sqrt{10} + \sqrt{10}$$
$$= 2\sqrt{17} + 2\sqrt{10}$$

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• use the midpoint formula to solve question 1 **OR** • plot the points and provide a geometrical solution for question 1 • provide a graphical or an algebraic solution for question 2 • plot the points, determine the midpoints graphically or algebraically, sketch the new quadrilateral, and attempt to find the distances for question 3.	• determine the distances algebraically, and determine the perimeter in simplified radical form for question 3.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.4 Solve problems involving rise, run and slope of line segments.
[PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 4.3.
- Problems involving slope also occur in Science 10.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• find the slope, given the graph of a line • find the slope, given any two points, using the slope formula • distinguish between negative and positive slopes • graph a line, given a point and the slope of the line • identify that vertical lines have an undefined slope and horizontal lines have a slope equal to zero.	• make connections between trigonometry and line segments • explain why vertical lines have an undefined slope and horizontal lines have a slope equal to zero • interpret slope in terms of rate or steepness • interpret the units for slope • interpret the meaning of slope on nonstandard grids.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.4 Solve problems involving rise, run and slope of line segments. [PS, V]

Sample Tasks

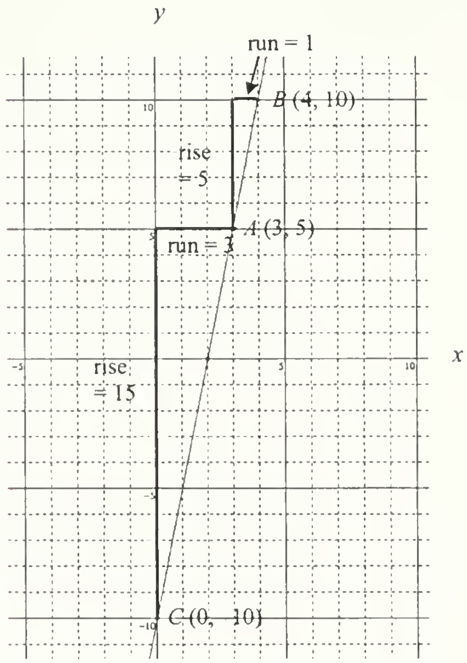
[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. Given $A(3, 5)$ is on a line of slope 5, find another point, B, on the line.

Solution:

1.



Go up 5 and over 1.
Use rise over run to find B graphically.
Another point could be (4, 10) or (5, 15), among others.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.4 Solve problems involving rise, run and slope of line segments. [PS, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

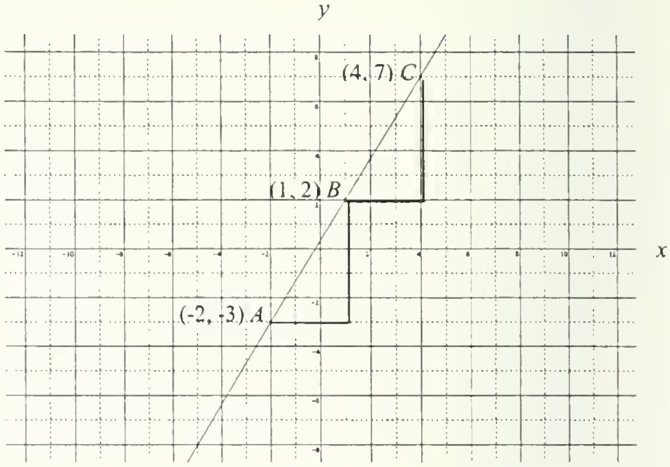
(continued)

Question:

2. Given $A(-2, -3)$, $B(1, 2)$ and $C(4, 7)$, verify that points A, B and C are collinear.

Solution:

2.



Count squares and state slope of $\overline{AB} = \frac{5}{3}$ and slope of $\overline{BC} = \frac{5}{3}$.

OR

$$\text{slope of } \overline{AB} = \frac{(2-(-3))}{(1-(-2))} = \frac{5}{3}$$

$$\text{slope of } \overline{BC} = \frac{(7-2)}{(4-1)} = \frac{5}{3}$$

OR

$$\overline{AB} : \frac{\text{rise}}{\text{run}} = \frac{5}{3}$$

$$\overline{BC} : \frac{\text{rise}}{\text{run}} = \frac{5}{3}$$

$\therefore$ A, B and C are collinear, as $\overline{AB}$ and $\overline{BC}$ have the same slope.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.4 Solve problems involving rise, run and slope of line segments. [PS, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Question:

3. Given a slope of $\frac{2}{3}$ and points $A(5, 7)$ and $B(x, 2)$, find the x-coordinate of point B.

Solution:

3. $(5, 7)$ and $(x, 2)$; $m = \frac{2}{3}$

$$\begin{aligned} \frac{2-7}{x-5} &= \frac{2}{3} \\ \frac{-5}{x-5} &= \frac{2}{3} \\ -15 &= 2(x-5) \\ -15 &= 2x-10 \\ -15+10 &= 2x \\ -5 &= 2x \\ -\frac{5}{2} &= x \end{aligned}$$

The coordinates of point B are $\left(-\frac{5}{2}, 2\right)$.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- plot point A on a coordinate plane and then use $\frac{\text{rise}}{\text{run}}$ to find the second point in question 1 - provide a complete solution for question 2 - plot points in question 3, use the slope to draw the line, and approximate the x-coordinate from the graph.	- determine the x-coordinate algebraically in question 3.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.5 Determine the equation of a line, given information that uniquely determines the line.
[PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 5.3.
- Students are not expected to solve linear systems in two variables; problems must be solvable without using systems.
- Linear systems are covered in Pure Mathematics 20.
- Students are expected to be familiar with the $y = mx + b$ and the $Ax + By + C = 0$ forms of the line. Vertical and horizontal lines are included.
- For the standard form $Ax + By + C = 0$, A, B and $C \in I$.
- It is optional to introduce linear regression using the graphing calculator to determine equations of lines.
- Students should include a sketch of the line as part of the solution, whenever they use a graphing calculator.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• identify the slope and y-intercept from $y = mx + b$ • write the equation in the form $y = mx + b$, given the slope and y-intercept • manipulate between the standard equation and slope–intercept form of line equations • recognize the equations of horizontal and vertical lines • determine the equation of a line, in slope–intercept form, given a point and the slope • determine the equation of a line, given a graph of the line with a labelled point on the line • determine the slope and find the equation of the line, given two points • determine the equation, even if a labelled point on the line is not given.	• solve real-world problems related to linear equations.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.5 Determine the equation of a line, given information that uniquely determines the line.
[PS, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. Given $A(6, 7)$ and $B(12, 14)$, determine the equation, in standard form, of the line passing through the two points.

Solution:

1.

$$\begin{aligned} m &= \frac{14-7}{12-6} = \frac{7}{6} \\ y &= \frac{7}{6}x + b \\ 7 &= \frac{7}{6}(6) + b \\ 7 &= 7 + b \\ 0 &= b \\ y &= \frac{7}{6}x + 0 \end{aligned}$$

$$\begin{aligned} y &= \frac{7}{6}x \\ 6y &= 7x \\ 6y - 7x &= 0 \\ 7x - 6y &= 0 \end{aligned}$$

Question:

2. Given a line with an x-intercept of 6 and a y-intercept of 2, determine the equation of the line.

Solution:

2. $(6, 0)$ $(0, 2)$
$x = 6$ $y = 2$

$$\begin{aligned} m &= \frac{2-0}{0-6} \\ &= \frac{2}{-6} \\ &= -\frac{1}{3} \\ m &= -\frac{1}{3} \end{aligned}$$

Therefore,

$$y = -\frac{1}{3}x + 2$$

or $x + 3y - 6 = 0$.

Students may also use linear regression on a graphing calculator with input points (6, 0) and (0, 2) to determine the equation of the line. In this case, the line will be expressed as $y = -0.33x + 2$.

(continued)

General and Specific Outcomes
General Outcome Solve coordinate geometry problems involving lines and line segments. **Specific Outcome** 3.5 Determine the equation of a line, given information that uniquely determines the line. [PS, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Question:

3. A student having a fixed amount of money can buy 6 cassettes and 6 CDs or 12 cassettes and 2 CDs. Assuming all cassettes are equally priced and all CDs are equally priced, determine the relationship between the number of cassettes, x, and number of CDs, y, the student can purchase.

Solution:

3. (6, 6) and (12, 2) are points.

$$\text{Slope } m = \frac{2-6}{12-6} = -\frac{2}{3}$$

Solution is $y - 2 = -\frac{2}{3}(x - 12)$

Alternative solutions can be in the forms:

$3y + 2x = 30$ or

$y = -\frac{2}{3}x + 10$

Question:

4. Write the equations of the horizontal line and the vertical line that pass through the point (2, –3).

Solution:

4. $x = 2$ (vertical); $y = -3$ (horizontal).

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• provide complete solutions for questions 1, 2 and 4.	• provide a complete solution for question 3.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.6 Solve problems using slopes of:
- parallel lines
- perpendicular lines.

[CN, PS, V]

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 4.4.
- Students are expected to be familiar with the $y = mx + b$ and the $Ax + By + C = 0$ forms of the line. Vertical and horizontal lines are included.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• determine if two lines are perpendicular or parallel, given their equations in standard form • determine if lines are parallel or perpendicular, given two points on each of the two lines • determine the equation of a line, given a point on the line and the equation of a parallel or perpendicular line • begin to find solutions to nonroutine problems.	• find solutions to nonroutine problems.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.6 Solve problems using slopes of:
- parallel lines
- perpendicular lines.

[CN, PS, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. Determine the equation of a line through a point (1, 5) and parallel to $3x - y + 5 = 0$.

Solution:

1. The line $3x - y + 5 = 0$ has a slope of 3.
   So $(y - 5) = 3\,(x - 1)$ is the equation of the required line.
   In simpler form, the equation is $y = 3x + 2$.

Question:

2. Determine the equation of a line, in standard form, that is perpendicular to the line $4x + 3y - 12 = 0$ and has the same x-intercept as $5x + 4y - 20 = 0$.

Solution:

2.
$$\begin{aligned} 4x + 3y - 12 &= 0 \\ 3y &= -4x + 12 \\ y &= -\frac{4}{3}x + 4 \\ \text{slope} &= -\frac{4}{3} \end{aligned}$$

The required line has a slope of $\frac{3}{4}$.

$5x + 4y - 20 = 0$
When $y = 0$, $x = 4$.

The required line passes through (4, 0).

The equation is:

$$y - 0 = \frac{3}{4}(x - 4)$$

or $$y = \frac{3}{4}x - 3$$

or $$3x - 4y - 12 = 0\,.$$

(continued)

General and Specific Outcomes
General Outcome Solve coordinate geometry problems involving lines and line segments. **Specific Outcome** 3.6 Solve problems using slopes of: • parallel lines • perpendicular lines. [CN, PS, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

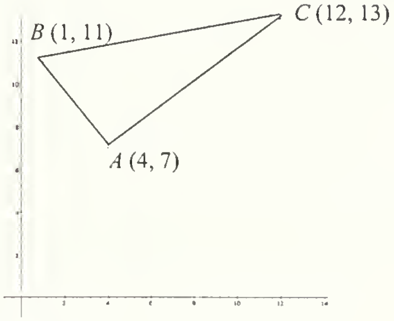
(continued)

Question:

3. A triangle has coordinates $A(4, 7)$, $B(1, 11)$ and $C(12, 13)$. Verify that ΔABC has a right angle at A, by using:
   a. the slopes of AB and AC
   b. Pythagorean theorem.

Solution:

3. a.



$$\text{slope } AB = \frac{11-7}{1-4} = -\frac{4}{3}$$

$$\text{slope } AC = \frac{13-7}{12-4} = \frac{3}{4}$$

These slopes satisfy $m_1 m_2 = -1$

So $AB \perp AC$.

b. $(AB)^2 = (1-4)^2 + (11-7)^2$
$= 25$
$(AC)^2 = (12-4)^2 + (13-7)^2$
$= 100$
$(BC)^2 = (13-11)^2 + (12-1)^2$
$= 125$

So $(BC)^2 = (AB)^2 + (AC)^2$, and $A = 90°$.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

3.6 Solve problems using slopes of:
- parallel lines
- perpendicular lines.

[CN, PS, V]

Sample Tasks

☐ Conceptual
☐ Procedural
☑ Problem-solving

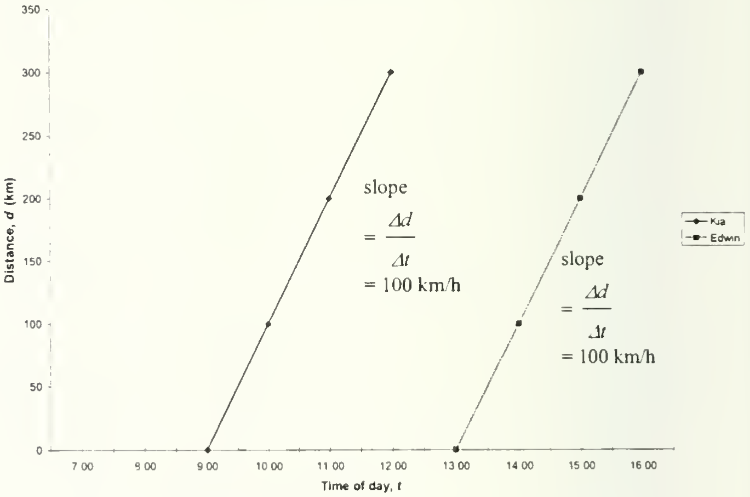
(continued)

Question:

4. Kia left Edmonton at 9:00 a.m. to drive to Calgary, a distance of 300 km. Edwin also drove from Edmonton to Calgary that day but left at 1:00 p.m. Each travelled at an average speed of 100 km/h. Graph their trips, using time of day on the horizontal axis and distance on the vertical axis.
   a. Show that the graphs are parallel lines.
   b. Explain how the relationship between the graphs would change if Edwin's average speed had been 105 km/h instead of 100 km/h.

Solution:

4. **Tracking Drivers**



a. The slopes are equal; therefore, the lines are parallel.

b. The lines would no longer be parallel, because the slopes would be 100 km/h and 105 km/h.

(continued)

General and Specific Outcomes
General Outcome Solve coordinate geometry problems involving lines and line segments. **Specific Outcome** 3.6 Solve problems using slopes of: • parallel lines • perpendicular lines. [CN, PS, V]

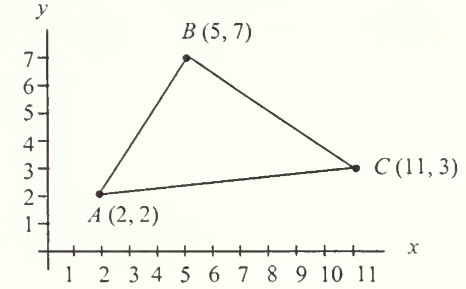
Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Question:

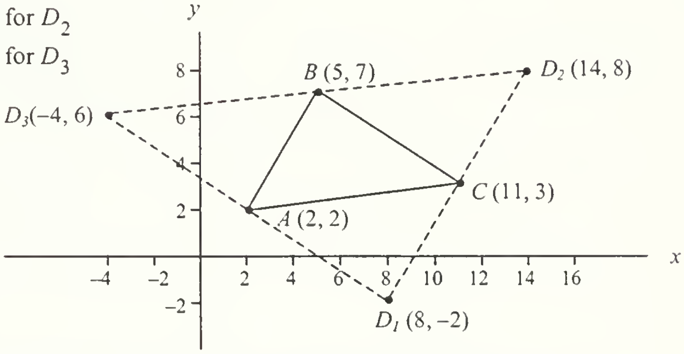
5. Use the following sketch of a graph to answer the question.



With A, B, C as indicated, find a fourth point D so that $ABCD$ represents a parallelogram. Justify your answer.

Solution:

5. (8, –2) for D_1
or (14, 8) for D_2
or (–4, 6) for D_3



Justification

For D_1, AD_1 is parallel to BC and equal in length to BC, so $ABCD_1$ is a parallelogram.
Following the convention of naming around a polygon, (8, –2) for D_1 is a solution. Points D_2 and D_3 form a parallelogram but the naming scheme does not follow the convention.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• provide complete solutions to questions 1, 2, 3 and 4 • find one of D_1 D_2 or D_3 in question 5, but provide little or no justification.	• link D to the conventional naming scheme for polygons • find two or more points for D in question 5, and provide justification with a diagram, or by linking line slopes and/or lengths to the properties of the parallelogram.

STANDARDS IN RELATIONS AND FUNCTIONS

GENERAL OUTCOMES

- Examine the nature of relations with an emphasis on functions.
- Represent data, using linear function models.
- Generate and analyze number patterns.

SPECIFIC OUTCOMES

4.1 Represent data, using function models. [CN, PS, V]

4.2 Use a graphing tool to draw the graph of a function from its equation. [C, T, V]

4.3 Describe a function in terms of:
- ordered pairs
- a rule, in word or equation form
- a graph.

[C, CN, V]

4.4 Use function notation to evaluate and represent functions. [C, PS]

4.5 Determine the domain and range of a relation from its graph. [PS, V]

4.6 Determine the following characteristics of the graph of a linear function, given its equation:
- intercepts
- slope
- domain
- range.

[PS, V]

4.7 Use direct variation, partial variation, and arithmetic sequences as applications of linear functions. [CN, PS, V]

4.8 Relate arithmetic sequences to linear functions defined over the natural numbers. [CN]

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.1 Represent data, using function models. [CN, PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 3.2.
- Equations for graphs are not necessary for this specific outcome.
- Students are expected to recognize graphs from scenarios and provide scenarios for graphs.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• generate sketches of graphs given a real-life relationship between variables • describe a real-life matching situation for graphs of functions, by stating the meanings of any intercepts, slopes, maxima and/or minima.	• incorporate quantification into the characteristics and descriptions of graphs.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.1 Represent data, using function models. [CN, PS, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

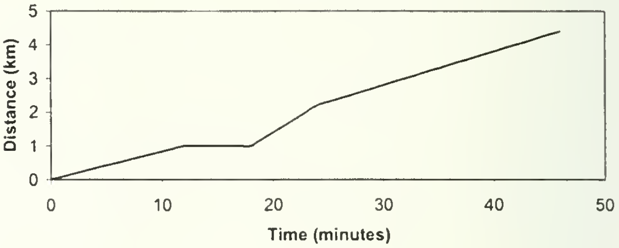
Question:

1. Sketch the graph of a woman walking her dog on her lunch hour. Use total distance as a function of time. Be creative with your graph. Write the corresponding story for your graph.

Solution:

1. (One of many possible solutions.)

Describing a Dog Walk

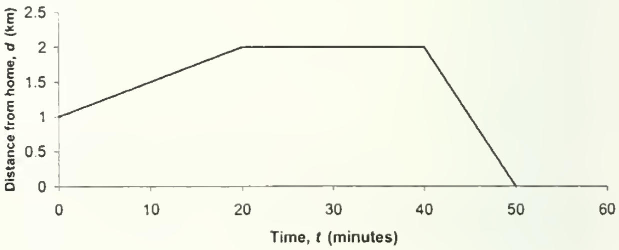


This dog walk shows 12 minutes of walking at a steady, slow pace, covering a distance of 1.0 km. Then there was a resting period, followed by 1.2 km at a faster pace. The return distance of 2.2 km was covered at a moderate pace. The total walking time was 46 minutes.

Question:

2. The graph below represents a jogging record. Compare the speed and direction of travel before and after the rest period.

Jogging Record



Solution:

2. Before rest: Travelling away from home at a rate of 1 km/20 min or 3 km/h.
After rest: Travelling toward home at a rate of 2 km/10 min or 12 km/h.

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.1 Represent data, using function models. [CN, PS, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

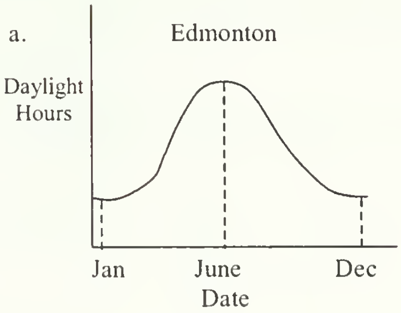
(continued)

Question:

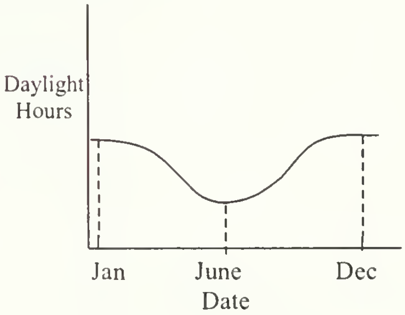
3. a. Sketch the graph of the number of daylight hours as a function of calendar date for Edmonton, Alberta.
   b. How does the graph change for Sydney, Australia?

Solution:

3. a.



b. Sydney



Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• sketch the graph in question 1, and write the story in a general way • provide an answer to question 2, if prompts are given • complete the graph in question 3, part a.	• provide scales for the axes, and quantify the changes in the graph in question 1 • provide an answer to question 2, without prompts • adapt the graph of question 3, part a, for Sydney.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.2 Use a graphing tool to draw the graph of a function from its equation. [C, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 3.3.
- This specific outcome is designed to introduce students to the appropriate use of a graphing tool.
- It is not specified if a graphing calculator or a computer is to be used. Most examples are given in terms of a graphing calculator.
- Students are expected to manipulate linear equations.
- The skill of isolating y is a prerequisite for using a graphing calculator.
- A thorough discussion of window settings is required.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• graph functions, using a graphing tool such as a graphing calculator or a computer graphing program • manipulate window parameters so a graph can be seen.	• manipulate linear equations having nonintegral coefficients, so a form of the equation is produced that can be entered into a calculator or graphing program.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.2 Use a graphing tool to draw the graph of a function from its equation. [C, T, V]

Sample Tasks

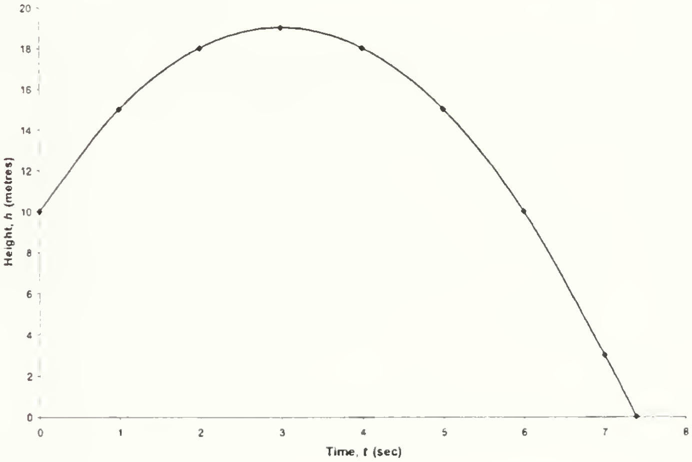
[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. An object was thrown and followed a path according to the equation $h = -t^2 + 6t + 10$, where h is the height in metres and t is the time in seconds.
   a. Using the following window settings, sketch the graph: $x_{\text{min}} = 0$, $x_{\text{max}} = 8$, $y_{\text{min}} = 0$, $y_{\text{max}} = 20$.
   b. Trace the path, using the trace function on a graphing calculator, and describe the function at key points.
   c. Are the window settings appropriate? Explain.

Solution:

1. a. **Height of a Thrown Object as a Function of Time**



   b. The graph begins at 10 m, the height from which the object is thrown. The object reaches a maximum height of 19 m, then falls to the ground before 8 seconds have passed.
   c. The window settings are appropriate; x represents the time, which begins at 0 and ends at 8 s, just after the object hits the ground; y represents the height, which never exceeds 20 m.

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.2 Use a graphing tool to draw the graph of a function from its equation. [C, T, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

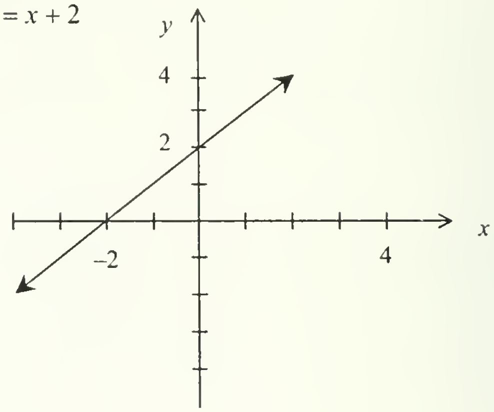
(continued)

Question:

2. Given the equation $x = y - 2$:
   a. Manipulate the above equation so it can be entered into a graphing calculator.
   b. Graph the function. Sketch the graph, providing appropriate window settings.

Solution:

2. a. $y = x + 2$
   b.



One possible solution may be to have the following window settings: $x_{\text{min}} = -4$, $x_{\text{max}} = +4$, $y_{\text{min}} = -4$, $y_{\text{max}} = +4$ (referring to the above sketch).

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• draw the graph in question 1 • provide appropriate answers for all parts of question 2.	• describe the graph in question 1 • justify the window settings in question 1.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.3 Describe a function in terms of:
- ordered pairs
- a rule, in word or equation form
- a graph.

[C, CN, V]

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 3.4.
- The choice of tests, such as the vertical line test, to determine if a relation is a function, is left up to the teacher.
- The use of the table feature on a graphing calculator is an important skill for this specific outcome.
- Relations can be described in terms of input and output values.
- Determining if an equation represents a function rather than a relation would be an extension of this specific outcome.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	There are no descriptions for the standard of excellence related to this specific outcome.
• make a table of values, list ordered pairs, sketch a graph, and write a rule in words for a given function, using an equation • determine if a relation is a function, given a: – graph and using the vertical line test – set of ordered pairs.	

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.3 Describe a function in terms of:
- ordered pairs
- a rule, in word or equation form
- a graph.

[C, CN, V]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. The cost of publishing books includes an initial cost of $2000 and a cost of $20 per book thereafter.
   a. Make a table of values for up to 1000 books.
   b. List all ordered pairs from your table of values.
   c. Graph the relation.
   d. Write an equation that represents this relation.
   e. Does this relation represent a function? Explain.

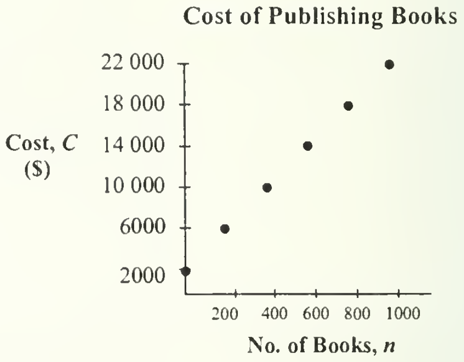
Solution:

1. a.

no. of books	0	200	400	600	800	1000
cost	$2000	$6000	$10 000	$14 000	$18 000	$22 000

   b. (0, 2000), (200, 6000), (400, 10 000), (600, 14 000), (800, 18 000), (1000, 22 000)

   c.



   d. $C = 20n + 2000$, where C is the total cost and n is the number of books.

   e. Yes, it does represent a function. In the graph, for every x value there is only one y value.

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.3 Describe a function in terms of:
- ordered pairs
- a rule, in word or equation form
- a graph.

[C, CN, V]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

2. Use the following tables of values to answer the question.

Table A

x	y
1	2
2	3
2	4

Table B

x	y
1	2
2	2
3	4

Which of the above represents a function and which does not? Explain your answer.

Solution:

2. Table B represents a function, because for each value of x there is only one value of y. In Table A there are two values of y when $x = 2$.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	There are no descriptions for the standard of excellence related to these sample tasks.
• provide a solution to all parts of question 1 and to question 2.	

General and Specific Outcomes
General Outcome Examine the nature of relations with an emphasis on functions. **Specific Outcome** 4.4 Use function notation to evaluate and represent functions. [C, PS]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 3.5.
- The operations of adding, subtracting, multiplying and dividing functions are included in Pure Mathematics 20.
- Composition of functions is also included in Pure Mathematics 20. The usual form of function notation is the form $f(x)$, where x can be a real number, a monomial or a binomial.
- Function notation can include spreadsheet formula notation.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• evaluate and represent functions with rational number and monomial inputs • use a graph or the table function to interpret notation.	• evaluate and represent functions with real number and binomial inputs.

General and Specific Outcomes
General Outcome Examine the nature of relations with an emphasis on functions. **Specific Outcome** 4.4 Use function notation to evaluate and represent functions. [C, PS]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Given $f(x) = 2x^2 + 3x - 6$, determine:

a. $f(2)$

Solution:

$$\begin{aligned} f(2) &= 2(2)^2 + 3(2) - 6 \\ &= 8 + 6 - 6 \\ &= 8 \end{aligned}$$

b. $f\left(\frac{3}{5}\right)$

Solution:

$$\begin{aligned} f\left(\frac{3}{5}\right) &= 2\left(\frac{3}{5}\right)^2 + 3\left(\frac{3}{5}\right) - 6 \\ &= \frac{18}{25} + \frac{9}{5} - 6 \\ &= \frac{18 + 45 - 150}{25} \\ &= -\frac{87}{25} \end{aligned}$$

c. $f(3t)$

Solution:

$$\begin{aligned} f(3t) &= 2(3t)^2 + 3(3t) - 6 \\ &= 18t^2 + 9t - 6 \end{aligned}$$

d. $f(\sqrt{5})$

Solution:

$$\begin{aligned} f(\sqrt{5}) &= 2(\sqrt{5})^2 + 3(\sqrt{5}) - 6 \\ &= 2(5) + 3\sqrt{5} - 6 \\ &= 10 + 3\sqrt{5} - 6 \\ &= 4 + 3\sqrt{5} \end{aligned}$$

e. $f(2t - 1)$

Solution:

$$\begin{aligned} f(2t-1) &= 2(2t-1)^2 + 3(2t-1) - 6 \\ &= 2(4t^2 - 2t - 2t + 1) + 6t - 3 - 6 \\ &= 2(4t^2 - 4t + 1) + 6t - 3 - 6 \\ &= 8t^2 - 8t + 2 + 6t - 9 \\ &= 8t^2 - 2t - 7 \end{aligned}$$

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.4 Use function notation to evaluate and represent functions. [C, PS]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

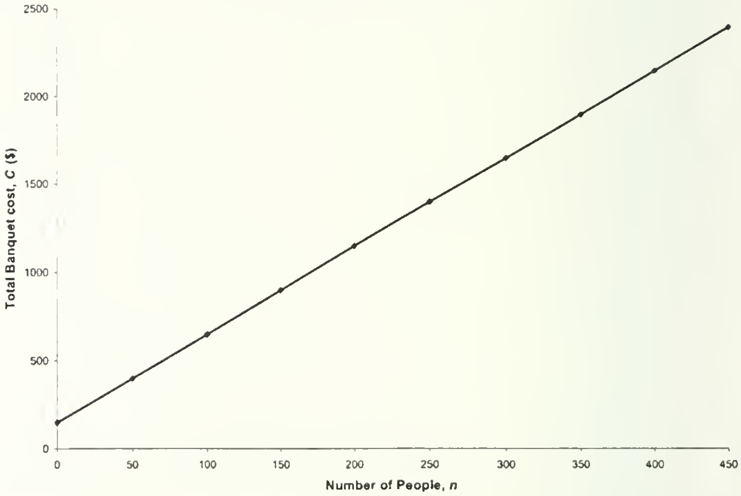
Question:

2. The cost for hosting a banquet is given by the formula $C(n) = 150 + 5n$, where C is the cost and n is the number of people.
   a. Use the graph or the table function to find $C(350)$.
   b. Verify algebraically the value of $C(350)$.
   c. $C(426) = 2280$. Interpret this statement.
   d. If $C(n) = 650$, find n.

Solution:

2. a. $C(350) = 1900$
   b. $C(350) = 150 + 5(350)$
   $= 150 + 1750$
   $= 1900$
   c. The cost of hosting a banquet for 426 people is $2280.
   d. Algebraic solution:
   $150 + 5n = 650$
   $5n = 500$
   $n = 100$
   Graphical solution:

Banquet Hosting Costs



Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• determine solutions to question 1, parts a, b and c • provide a solution to parts a, b and c of question 2.	• determine solutions to parts d and e of question 1 • provide a complete solution to question 2.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.5 Determine the domain and range of a relation from its graph. [PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 3.6.
- Set notations for domain and range are not required.
- Graphs must be easily drawn or provided. Students can be expected to draw the graph of a function of the form $y = f(x)$, following specific outcome 4.2.
- Students are expected to manipulate only linear equations.
- It is expected that students will use graphing calculator technology for this specific outcome.
- Teachers have the option of showing students how to use the trace function or the calculate function on a graphing calculator for determining range.
- Graphs of relations that are not functions must be provided. Examples are expressions such as $4x^2 + 9y^2 = 26$ or $3y + 4x^2 - 9x = 0$.
- Students should be encouraged to relate the concepts of domain and range to real-life situations.
- Domains and ranges for problem contexts may differ from the domains and ranges of the relevant algebraic functions.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• state the domain and range for most graphs • interpret the domain and range from word problems, with guidance.	• interpret the domain and range from graphs and from word problems, without guidance.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.5 Determine the domain and range of a relation from its graph. [PS, V]

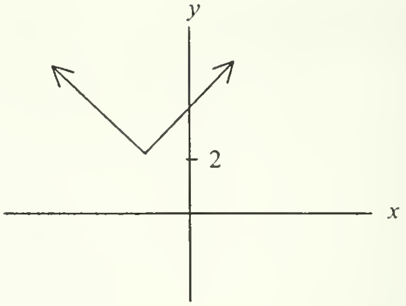
Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

Question:

1. Based on the following, state the domain and range.

$y = |x + 1| + 2$



Solution:

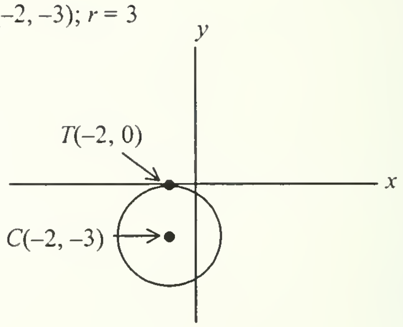
1. Domain: $x \in \Re$
Range: $y \geq 2$

Question:

2. Determine the domain and range of a circle with centre $(-2, -3)$ and a radius of 3. Draw the graph first.

Solution:

2. Centre, $C = (-2, -3)$; $r = 3$



Domain: $-5 \leq x \leq 1$
Range: $-6 \leq y \leq 0$

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.5 Determine the domain and range of a relation from its graph. [PS, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

(continued)

Question:

3. The relation between the distance driven, d, in km, and the cost, c, in dollars, of renting a car is given by $c = 45 + 0.20d$. Determine the domain and range of this relation.

Solution:

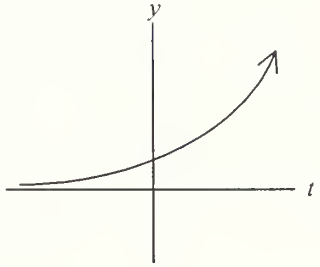
3. Domain: $d \geq 0$
   Range: $c \geq 45$

Question:

4. a. Sketch the graph of $y = 5000(1.05)^t$.
   b. State the domain and range of the graph obtained in a.
   c. Suppose the function $y = 5000(1.05)^t$ describes the growth of \$5000 at 5% compounded annually. State the domain and range of the function.
   d. Explain why the domain and range are different in parts b and c.

Solution:

4. a.



   b. Domain: all real numbers
      Range: $y > 0$
   c. Domain: $t \geq 0$
      Range: $A \geq 5000$
   d. In reality, time is never a negative value. Also the amount of money can never be less than \$5000.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• state the domain and range in questions 1, 2 and 3 • determine solutions to parts a and b in question 4.	• recognize and explain the discrete nature of the data in question 3; i.e., nearest kilometre and nearest cent • determine solutions to parts c and d in question 4.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.6 Determine the following characteristics of the graph of a linear function, given its equation:
- intercepts
- slope
- domain
- range.

[PS, V]

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 5.2.
- Students are not expected to solve linear systems in two variables; problems must be solvable without using systems.
- Linear systems are covered in Pure Mathematics 20.
- It is appropriate to emphasize the mathematical process of technology in this specific outcome.
- Students should be encouraged to rewrite equations into the $y = mx + b$ form and to use graphing tools in their analyses.
- It is assumed that specific outcome 3.4 has been covered previously.
- It is expected that the student can interpolate and extrapolate points on the graph of a given linear function.
- The concepts of domain and range can be related to everyday situations that have restrictions or boundaries; e.g., negative time is not meaningful, or there are limits on how high a person can throw a ball.
- Alternative names for the independent variable include *manipulated* and *input.*
- Alternative names for the dependent variable include *responding* and *output.*

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• determine the following characteristics, given an equation of a linear function in any form: intercepts, slope, domain and range.	• distinguish between discrete and continuous data, and explain how this may affect the domain, range and intercepts.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

4.6 Determine the following characteristics of the graph of a linear function, given its equation:
- intercepts
- slope
- domain
- range.

[PS, V]

Sample Task

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

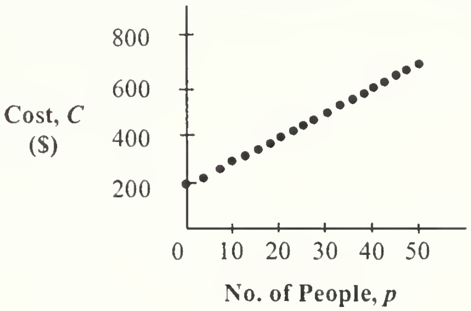
Question:

1. A basketball team is hosting a banquet. The caterer charges a set-up fee of $200 plus $10 per person. The equation $C = 200 + 10p$ represents the cost of the banquet.
   a. Sketch a graph, with the cost as a function of the number of people.
   b. Determine the cost for 30 people attending.
   c. If the total cost is to be no more than $700, how many people can attend?
   d. Determine the slope of the graph, and explain the significance of the slope in this situation.
   e. State the y-intercept, and explain the significance of the y-intercept in this situation.
   f. State the domain and range of the function, and explain why there may be restrictions on both.

Solution:

1. a.

Cost of Basketball Team's Banquet



b. $C = 200 + 10(30)$
$C = 200 + 300$
$C = \$500$

c. $700 = 200 + 10p$
$500 = 10p$
$p = 50$
The maximum number of people is 50.

(continued)

General and Specific Outcomes
General Outcome Examine the nature of relations with an emphasis on functions. **Specific Outcome** 4.6 Determine the following characteristics of the graph of a linear function, given its equation: • intercepts • slope • domain • range. [PS, V]

Sample Task

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:

1. d. slope = 10 or $10/person
   slope is cost/person
   e. y-intercept = 200
   y-intercept is the set-up fee for the banquet
   f. Domain: 0, 1, 2, … to capacity of hall
   Range: 200, 210, 220, … until hall is full
   There are domain restrictions, because you can never have a negative or fractional number of people, or more people than the banquet hall can hold. There are range restrictions, because the cost goes up in steps of $10 from $200 until the hall is full.

Descriptions of Student Performance (Related to Sample Task)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• graph the function • interpolate and extrapolate from the graph • determine the slope and the y-intercept • determine the domain as $x \geq 0$ and the range as $y \geq 200$ • explain the significance of the slope and y-intercept.	• restrict the domain to the capacity of the hall; determine the range as 200, 210, 220, …; and explain the restrictions on the domain and range.

General and Specific Outcomes

General Outcome

Represent data, using linear function models.

Specific Outcome

4.7 Use direct variation, partial variation, and arithmetic sequences as applications of linear functions. [CN, PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 5.4.
- It is appropriate to emphasize the use of technology in this specific outcome.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• determine the equation for a direct variation situation • determine the equation for a partial variation situation • recognize that direct and partial variation situations are represented by linear functions • graph direct and partial variation situations • determine x- and y-intercepts from a graph • determine slope • make a table of values • interpolate from a graph to make estimates • write the equation, based on a graph.	• explain the significance of the x- and y-intercepts of a graph • interpret slope • write the equation without using a table of values or a graph.

General and Specific Outcomes

General Outcome

Represent data, using linear function models.

Specific Outcome

4.7 Use direct variation, partial variation, and arithmetic sequences as applications of linear functions. [CN, PS, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. At a certain time of the day, the length of the shadow of a tree varies directly as the height of the tree. A 3.0 m tree casts a shadow of 1.2 m.
   a. Determine the constant of variation.
   b. Write the equation, representing the length of the shadow as a function of the height of the tree.
   c. Determine the length of a shadow of a 5.0 m tree.
   d. Determine the height of a tree that produces a shadow that is 6.8 m long.
   e. Graph the function.

Solution:

1. a. $L \propto h$

$L = kh$

$k = \frac{1.2}{3} \therefore k = 0.4$

b. $L = 0.4h$

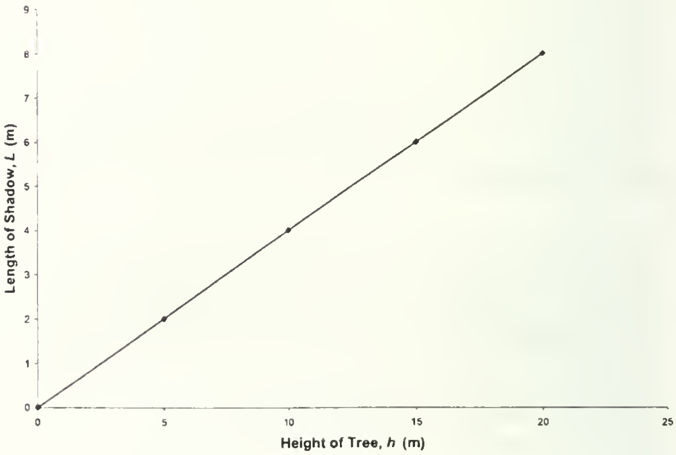
c. $L = (0.4)(5.0 \text{ m})$

$L = 2.0 \text{ m}$

d. $6.8 \text{ m} = 0.40h$

$h = 17 \text{ m}$

e. **Length of Shadow versus Height of Tree**



(continued)

General and Specific Outcomes

General Outcome

Represent data, using linear function models.

Specific Outcome

4.7 Use direct variation, partial variation, and arithmetic sequences as applications of linear functions. [CN, PS, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Question:

2. A vehicle's fuel tank has a maximum capacity of 90 L. Gasoline consumption is rated at 15 L/100 km for the vehicle.
   a. Make a table of values for the distance driven and the amount of fuel left in the tank.
   b. Write the equation, using the amount remaining in the tank as a function of the distance driven.
   c. Graph the function.
   d. Use the graph to estimate the distance travelled if 50 L of fuel remain in the tank; verify this estimate, using your equation.
   e. What distance has been covered when the fuel tank is empty, and how does this relate to the graph?
   f. Explain the significance of the y-intercept.
   g. Explain the significance of the slope.

Solution:

2. a.

Distance (km)	0	100	200	300	400	500	600
Gasoline (L)	90	75	60	45	30	15	0

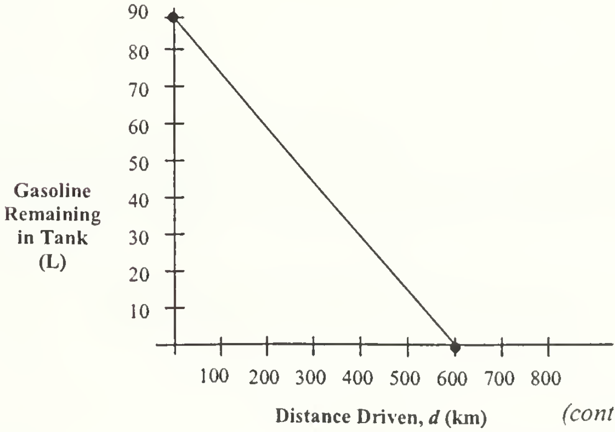
b. $g = md + b$

$$m = \frac{75-90}{100-0} = \frac{-15}{100} = -0.15$$

$b = 90$ (from table)

$\therefore g = -0.15d + 90$

c. **Gasoline Remaining in Tank versus Distance Driven**



(continued)

General and Specific Outcomes
General Outcome Represent data, using linear function models. **Specific Outcome** 4.7 Use direct variation, partial variation, and arithmetic sequences as applications of linear functions. [CN, PS, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)

Solution:

2. d. Estimate = 260 km (from graph)

$$\begin{aligned} 50 &= -0.15d + 90 \\ -40 &= -0.15d \\ -4000 &= -15d \\ d &= 267 \text{ km} \end{aligned}$$

e. 600 km–this is represented by the x-intercept of the graph.
f. The y-intercept represents the maximum capacity of the tank.
g. The slope represents the gasoline consumption in L/km.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• provide a complete solution for question 1 • provide a solution for question 2, parts b, c, d and e, given the table in part a.	• provide a complete solution for question 2.

General and Specific Outcomes

General Outcome

Generate and analyze number patterns.

Specific Outcome

4.8 Relate arithmetic sequences to linear functions defined over the natural numbers. [CN]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Both step and point functions are included in this specific outcome. Teachers may want to address the specific outcomes related to $y = mx + b$, slope and y-intercept notation prior to addressing this specific outcome.
- Students are not expected to solve linear systems in two variables; problems must be solvable without using systems. Linear systems are covered in Pure Mathematics 20.
- Students should be able to differentiate between graphs for discrete and continuous data.
- This specific outcome is limited to discrete data.
- The use of graphing calculator technology is appropriate for this specific outcome.
- Students need to know how the y-intercept of the linear function relates to the sequence.
- The emphasis should be on the sequence formula and the use of $y = mx + b$ notation.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• generate the graph of a linear equation that represents an arithmetic sequence • determine a linear function from a table, a graph or a description.	• recognize the discrete nature of data and the significance of the y-intercept as it relates to an arithmetic sequence.

General and Specific Outcomes

General Outcome

Generate and analyze number patterns.

Specific Outcome

4.8 Relate arithmetic sequences to linear functions defined over the natural numbers. [CN]

Sample Task

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

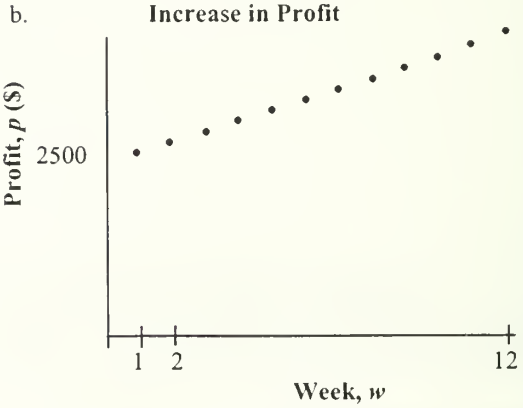
1. In preparation for the holiday season, a chocolate factory introduces a new product. After the first week, the profit was $2500. The profits increase by $45 per week for the 12 weeks before the holiday season.
   a. Make a table of values for profit versus weeks for the 12 weeks.
   b. Graph the relationship.
   c. Determine if the relationship is linear or nonlinear.
   d. Explain why the domain is the set of natural numbers.
   e. Determine if the range is an arithmetic sequence.
   f. Write an expression for the relationship in terms of a linear function.
   g. Explain why the graph has no intercept on the vertical axis.

Solution:

1. a.

Week	Profit ($)
1	2500
2	2545
3	2590
4	2635
5	2680
6	2725
7	2770
8	2815
9	2860
10	2905
11	2950
12	2995

b.



c. The relationship is linear.

(continued)

General and Specific Outcomes

General Outcome

Generate and analyze number patterns.

Specific Outcome

4.8 Relate arithmetic sequences to linear functions defined over the natural numbers. [CN]

Sample Task

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Solution:

d. The domain is the set of natural numbers, because parts of a week, zero weeks or negative weeks, do not exist.

e. The range is 2500, 2545, 2590, …, and the difference is always 45, so the range is an arithmetic sequence.

f.
$$p = mw + b$$
$$\text{Slope} = \frac{2545 - 2500}{2 - 1}$$
$$= 45$$
y-intercept (b)
$$2500 = 45(1) + b$$
$$b = 2455$$
$$\therefore p = 45w + 2455$$

g. The profit is undefined at 0 weeks.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can: • make a table of values in part a • graph the relationship in part b • recognize that the relationship is linear • determine the range as an arithmetic sequence in part e • write the linear relationship in part (i).	**A student demonstrating the *standard of excellence* can also:** • explain the significance of the domain, and realize that the data are discrete in part d • recognize, in part g, that there will be no y-intercept, since the number of weeks must be natural numbers (t_o does not exist).

STANDARDS IN EXPONENTS AND RADICALS

GENERAL OUTCOMES

- Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.
- Use basic arithmetic operations on real numbers to solve problems.
- Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.

SPECIFIC OUTCOMES

5.1 Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. [C, R, V]

5.2 Use approximate representations of irrational numbers. [R, T]

5.3 Communicate a set of instructions used to solve an arithmetic problem. [C]

5.4 Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T]

5.5 Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E]

5.6 Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]

Strand: Number (Number Concepts)

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

General and Specific Outcomes

General Outcome

Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

Specific Outcome

5.1 Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system.
[C, R, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 2.3.
- Classification of numbers includes criteria for inclusion of a number into the appropriate subset of the real number system.
- This topic was introduced in Grade 9 Mathematics.
- Students should be familiar with how to use their graphing calculators to convert a decimal to a fraction, but they also need to be able to do this algebraically.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• describe the different number sets • categorize numbers into the appropriate number set • explain how a number can belong to more than one number set	• describe how to convert a repeating decimal into a fraction algebraically.

General and Specific Outcomes

General Outcome

Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

Specific Outcome

5.1 Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system.
[C, R, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

Question:

1. a. Classify the following numbers into the appropriate number set: $8,\ \sqrt{5},\ 0,\ \frac{5}{9},\ -6$.

natural ____________
whole ____________
integer ____________
rational ____________
irrational ____________

b. Explain why 8 belongs to four number sets.

Solution:

1. a.

natural	8
whole	0 and 8
integer	−6, 0 and 8
rational	$\frac{5}{9}$, −6, 0 and 8
irrational	$\sqrt{5}$

b. The number 8 is a natural number, so must also be a whole number and an integer. It is also a rational number as it can be written as the fraction $\frac{8}{1}$.

Question:

2. Use a calculator to determine the decimal representations of $\frac{13}{5}$, $\frac{32}{99}$, $\sqrt{3}$ and π. Why are $\sqrt{3}$ and π described as irrational, while $\frac{13}{5}$ and $\frac{32}{99}$ are described as rational? Use your rule to determine if $\sqrt{0.614656}$ is rational or irrational.

Solution:

2. $\frac{13}{5} = 2.6$, $\frac{32}{99} = 0.3232\ldots$

$\sqrt{3} = 1.732050808\ldots$, $\pi = 3.141592654\ldots$

$\frac{13}{5}$ is a terminating decimal; $\frac{32}{99}$ is a repeating decimal. Both are rational. $\sqrt{3}$ and π neither terminate nor repeat. They are both irrational.

$\sqrt{0.614656} = 0.784$, which is a terminating decimal, so $\sqrt{0.614656}$ is rational.

(continued)

General and Specific Outcomes
General Outcome Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system. **Specific Outcome** 5.1 Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. [C, R, V]

Sample Tasks

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

(continued)

Question:

3. Explain why all integers are also rational numbers and why no rational numbers are irrational.

Solution:

3. An integer can be written as $\frac{p}{1}$, so is of the form $\frac{p}{q}$, and is a rational number.

   Rational numbers are either repeating decimals or terminating decimals. Irrational numbers have non-repeating and non-terminating decimals. A number cannot be repeating and non-repeating at the same time. Nor can a number be terminating and non-terminating at the same time.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• classify the numbers in question 1 • explain why 8 belongs to four different number sets in question 1, part b • find the decimal representations of the numbers in question 2 • identify a rule in question 2 to separate $\frac{13}{5}$ and $\frac{32}{99}$ from $\sqrt{3}$ and π • give approximate explanations of the statements in question 3.	• apply the rule in question 2 to show that $\sqrt{0.614656}$ is rational • give precise explanations for the statements in question 3.

General and Specific Outcomes

General Outcome

Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

Specific Outcome

5.2 Use approximate representations of irrational numbers. [R, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 2.4.
- Both mental estimates and calculator approximations are appropriate for this specific outcome.
- Students must be able to use their calculators to find the n^{th} root of a number.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• estimate the square root of a two-digit number to the nearest whole number, without using a calculator • use approximate values of irrational numbers to rank them • use technology to verify approximations • round correctly to a given number of decimal places.	• estimate the cube root of a two-digit number to the nearest whole number, without using a calculator • select an appropriate approximation, and describe why one approximation may be better than another • place irrational numbers on a real number line.

General and Specific Outcomes

General Outcome

Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

Specific Outcome

5.2 Use approximate representations of irrational numbers. [R, T]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. Two sides of a triangle are 2 cm and 3 cm. Harriet has determined that the length of the third side is $5\sqrt{2}$ cm. Is her answer reasonable? Why or why not?

Solution:

1. $5\sqrt{2}$ cm is approximately 7.07 cm. This answer is not reasonable, because the third side must be shorter than 5 cm—shorter than the sum of the other two sides.

Question:

2. Use estimates to explain why $\sqrt{3} + \sqrt{4} + \sqrt{5}$ is not equal to $\sqrt{12}$.

Solution:

2. $\sqrt{3}$ is between 1 and 2, $\sqrt{4}$ is equal to 2, and $\sqrt{5}$ is between 2 and 3. So $\sqrt{3} + \sqrt{4} + \sqrt{5}$ is between 5 and 7. $\sqrt{12}$, on the other hand, is less than 4.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• determine the approximate value for $5\sqrt{2}$ in question 1 • use calculator values, or rounded calculator values, to judge the conjecture in question 2.	• explain why $5\sqrt{2}$ is not reasonable in question 1, by comparing $5\sqrt{2}$ to 5 in the context of the problem • use estimates and compare the estimated values of $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$ and $\sqrt{12}$ to judge the conjecture in question 2.

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

5.3 Communicate a set of instructions used to solve an arithmetic problem. [C]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 2.5.
- Students may have to review order of operations.
- Students should be able to communicate keying instructions that follow the algebraic format of the formula.
- Students should be made aware that the expression or formulas may sometimes need reformatting, using parentheses, before the keying sequence will work.
- Students should gain experience in reformatting expressions or formulas.
- Keying sequences should be covered in the appropriate problem contexts.
- The instruction sets are not limited to calculator keying sequences.
- Teachers may introduce number operations as concepts that are developed throughout the course; e.g., the concept of absolute value may be developed and applied in the coordinate geometry outcomes—specific outcomes 3.2–3.6.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• give the set of instructions for a simple arithmetic expression • evaluate an arithmetic expression involving one set of absolute value symbols.	• give the set of instructions for complex arithmetic expressions • explain the reason for the order of keystrokes.

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

5.3 Communicate a set of instructions used to solve an arithmetic problem. [C]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. a. Given the formula for the surface area of a cylinder, $SA = 2\pi r(r + h)$, write a set of instructions that will allow another student to use the formula when the radius is 5 cm and the height is 12 cm.
   b. Create a set of keystroke instructions for using technology to find the volume of a sphere with a radius of 5 cm.

Solution:

1. a.
   - add the radius and the height
   - multiply the result by the radius
   - multiply the result by 2π

   OR

   $2 \times$ 2nd $\pi \times 5 \times (5 + 12)$ ENTER

   ↑ may be omitted

   ↑ or = on many calculators

   b. Standard formula:

   $$V = \frac{4}{3}\pi r^3$$

   $4 \times$ 2nd$\pi \times 5 \wedge 3 \div 3$ ENTER

   ↑ or = on many calculators

   OR

   $5 \wedge 3 \times 4 \times$ 2nd $\pi \div 3$ ENTER

   ↑ or = on many calculators

Question:

2. Create a set of keystrokes that will evaluate A, where

$$\cos A = \left[\frac{2^2 + 3^2 - 2.5^2}{2(2)(3)}\right].$$

Solution:

2. One of many solutions:

   2nd cos $((2 \wedge 2 + 3 \wedge 2 - 2.5 \wedge 2) \div (2{*}2{*}3))$ ENTER

   ↑ or = on many calculators

(continued)

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

5.3 Communicate a set of instructions used to solve an arithmetic problem. [C]

Sample Tasks

☐ Conceptual
☑ Procedural
☐ Problem-solving

(continued)

Question:

3. Here is an example of a number puzzle.
   - Choose a whole number.
   - Add 5.
   - Double the result.
   - Subtract 4.
   - Divide the result by 2.
   - Subtract the number you started with.
   - The result is 3.

   Does this number puzzle work if you:
   - choose a negative integer?
   - choose a fraction?

   Create a puzzle of your own that is between 5 and 10 lines in length and that has a final result of 2.

Solution:

3. Using a Negative Integer

Start with −10; then	add 5 to get	−5
	double to get	−10
	subtract 4 to get	−14
	divide by 2 to get	−7
	subtract −10 to get	3

Using a Fraction

Start with $\frac{3}{5}$; then	add 5 to get	$\frac{28}{5}$
	double to get	$\frac{56}{5}$
	subtract 4 to get	$\frac{36}{5}$
	divide by 2 to get	$\frac{18}{5}$
	subtract $\frac{3}{5}$ to get	3

Creating a New Puzzle

Answers will vary.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• write a set of instructions or a set of keystrokes that will solve formulas similar to those in question 1 • verify that the puzzle in question 3 works for a negative integer or a fraction • create a new puzzle, in question 3, that is simple and similar to the one given.	• rewrite formulas, such as the one in question 2, into a form that is compatible with the calculator being used • write a set of keystrokes that allow a user to calculate a value for A in formulas such as the one in question 2 • create a new puzzle, in question 3, that is creative and complex.

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

5.4 Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 2.6.
- For this specific outcome, precise calculations are done on the calculator, while estimates can be done mentally. Computations using exact values are part of specific outcome 5.6.
- Students need to be reminded that decimal values should not be rounded until the end of a calculation.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• perform arithmetic operations on irrational numbers, using a calculator.	• give a reasonable estimate of an answer, without using a calculator, and use the estimate to check the keying sequence of the calculator solution • evaluate expressions involving more than one radical symbol; e.g., $\sqrt{\sqrt{32}}$.

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

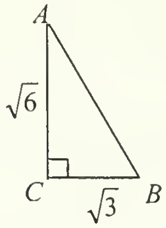
5.4 Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Using the Pythagorean formula, $c^2 = a^2 + b^2$, and given the following triangle:



   a. use a two decimal approximation for the sides of the triangle and solve for the missing side; express the length of the missing side to four decimal places
   b. use the radical form for the sides of the triangle to solve for the missing side
   c. explain why the two answers are different.

Solution:

1. a. $c^2 = (1.73)^2 + (2.45)^2$, $c = 2.9992$

   b. $c^2 = \left(\sqrt{3}\right)^2 + \left(\sqrt{6}\right)^2$, $c = 3$

   c. One answer is a whole number and the other is a decimal. The decimal input values are precise to the hundredths place, and the irrational values are precise to an infinite number of decimal places.

(continued)

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

5.4 Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T]

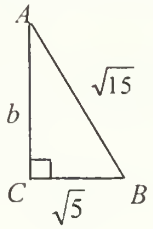
Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

2. Find the missing side, given the following triangle.



Solution:

2.

$$\begin{aligned}\left(\sqrt{15}\right)^2 &= \left(\sqrt{5}\right)^2 + b^2\\ b^2 &= \left(\sqrt{15}\right)^2 - \left(\sqrt{5}\right)^2\\ b &= 3.162 \text{ (inputting exact values)}\\ &\text{or } 3.155 \text{ (inputting two decimal place approximation).}\end{aligned}$$

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the ***acceptable standard*** **can:**	**A student demonstrating the** ***standard of excellence*** **can also:**
• determine the lengths of the missing sides for questions 1 and 2, using decimal approximations and radical form, when the first step is provided • start to write a comparison of the two answers in question 1, part c.	• determine the lengths of the missing sides for questions 1 and 2, even when the first step is not provided • provide a detailed and insightful comparison of the two answers in question 1, part c.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.

Specific Outcome

5.5 Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E]

Notes:

- The exponent laws with integral exponents are covered in Grade 9 Mathematics. However, numerical bases, rather than variable bases, are used in Grade 9.
- Proof of whether a particular number is rational or irrational can be included as part of Pure Mathematics 20.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• write a power with a positive rational exponent in equivalent radical form and vice versa.	• manipulate expressions containing more than one power with rational exponents • convert expressions containing more than one radical symbol with different indexes to powers with rational exponents • simplify expressions and convert back to radical form.

General and Specific Outcomes

General Outcome

Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.

Specific Outcome

5.5 Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Express $x^{\frac{2}{3}}$ as a radical.

Solution:

1. $\sqrt[3]{x^2}$ or $\left(\sqrt[3]{x}\right)^2$

Question:

2. Express $\sqrt[5]{x}$, using exponents.

Solution:

2. $x^{\frac{1}{5}}$

Question:

3. Find the exact value of $(-8)^{-\frac{2}{3}}$.

Solution:

3. $(-8)^{-\frac{2}{3}} = \left[(-2)^3\right]^{-\frac{2}{3}} = (-2)^{-2} = \dfrac{1}{(-2)^2} = \dfrac{1}{4}$

OR

$$(-8)^{-\frac{2}{3}} = \left(-\frac{1}{8}\right)^{\frac{2}{3}} = \left(\sqrt[3]{-\frac{1}{8}}\right)^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}.$$

Question:

4. Express $\sqrt{2} \times \sqrt[3]{2}$ as a single radical.

Solution:

4. $\sqrt{2} \times \sqrt[3]{2} = 2^{\frac{1}{2}} \times 2^{\frac{1}{3}} = 2^{\frac{5}{6}}$, so is equal to $\sqrt[6]{32}$ or $\sqrt[6]{2^5}$.

Question:

5. Sarah used her calculator to determine that $(-27)^{-\frac{2}{3}}$ is $\frac{1}{9}$. Use the exponent laws to show that this answer is correct.

Solution:

5.
$$\begin{aligned} -27 &= (-3)^3 \\ (-27)^{-\frac{2}{3}} &= \left[(-3)^3\right]^{-\frac{2}{3}} \\ &= [-3]^{-2} \\ &= \frac{1}{(-3)^2} \\ &= \frac{1}{9} \end{aligned}$$

(continued)

General and Specific Outcomes

General Outcome

Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.

Specific Outcome

5.5 Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E]

Sample Tasks

☐ Conceptual
☑ Procedural
☐ Problem-solving

(continued)

Question:

6. Express the simplified form of $\left(\sqrt[6]{a^5}\right)\left(\sqrt[3]{a^2}\right)$ as a:
   a. power
   b. radical.

Solution:

6. a. $\left(a^{\frac{5}{6}}\right)\left(a^{\frac{2}{3}}\right)=a^{\left(\frac{5}{6}+\frac{2}{3}\right)}=a^{\frac{3}{2}}$

   b. $\left(a^{\frac{5}{6}}\right)\left(a^{\frac{2}{3}}\right)=a^{\left(\frac{5}{6}+\frac{2}{3}\right)}=a^{\frac{3}{2}}=\sqrt{a^3}$

Question:

7. Express the simplified form of $\sqrt{\sqrt[3]{x^5}}$ as a:
   a. power
   b. radical.

Solution:

7. a. $\left[x^{\frac{5}{3}}\right]^{\frac{1}{2}}=x^{\frac{5}{6}}$

   b. $\sqrt[6]{x^5}$

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• provide answers for questions 1 and 2 • write $\sqrt[6]{a^5}$ and $\sqrt[3]{a^2}$ using exponents in question 6.	• answer questions 3, 4, 5, 6 and 7.

General and Specific Outcomes

General Outcome

Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.

Specific Outcome

5.6 Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Examples using the exact values of the sines, cosines and tangents of special angles are part of Pure Mathematics 30.
- Examples using triangles inscribed in circles are included in Pure Mathematics 20.
- Although programs for conversion of radicals from mixed to entire (and vice versa) are available, these programs should be deleted from students' calculators before any written assessment.
- This specific outcome cannot be assessed if students are allowed to use calculators with symbol manipulation capabilities.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• convert mixed radicals to entire radicals and vice versa—square roots only and limiting radicands to natural numbers • perform the operations of addition and subtraction with square roots only and whole number radicands less than 100 • determine the product of: – two mixed or entire radicals – a mixed radical and a binomial involving radicals – two binomials involving entire radicals only • divide a monomial radical by a monomial radical.	• convert mixed radicals to entire radicals and vice versa, including radicals with cube roots and variables • perform the operations of addition and subtraction with all square roots and those cube roots involving easily recognized values • determine the product of: – two binomials involving mixed or entire radicals – the square of a mixed radical and a binomial involving radicals • divide a binomial radical by a binomial radical.

General and Specific Outcomes

General Outcome

Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.

Specific Outcome

5.6 Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Express $\sqrt{50}$ as a simplified radical.

Solution:

1. $\sqrt{50} = \sqrt{25}\sqrt{2}$
   $\sqrt{50} = 5\sqrt{2}$

Question:

2. Express $3\sqrt{6}$ as an entire radical.

Solution:

2. $3\sqrt{6} = \sqrt{9}\sqrt{6}$
   $= \sqrt{54}$

Question:

3. Express $5a\sqrt{f}$ as an entire radical.

Solution:

3. $5a\sqrt{f} = \sqrt{25}\sqrt{a^2}\sqrt{f}$
   $= \sqrt{25a^2 f}$

Question:

4. Simplify $\sqrt{18} + \sqrt{32}$.

Solution:

4. $\sqrt{18} + \sqrt{32} = \sqrt{9}\sqrt{2} + \sqrt{16}\sqrt{2}$
   $= 3\sqrt{2} + 4\sqrt{2}$
   $= 7\sqrt{2}$

(continued)

General and Specific Outcomes

General Outcome

Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.

Specific Outcome

5.6 Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

5. Simplify $\sqrt[3]{128} + 4\sqrt[3]{16}$.

Solution:

5. Either:
$\sqrt[3]{128} + 4\sqrt[3]{16}$
$= \sqrt[3]{8}\,\sqrt[3]{16} + 4\sqrt[3]{16}$
$= 2\sqrt[3]{16} + 4\sqrt[3]{16}$
$= 6\sqrt[3]{16}$
OR:
$\sqrt[3]{128} + 4\sqrt[3]{16}$
$= \sqrt[3]{64}\,\sqrt[3]{2} + 4\sqrt[3]{8}\,\sqrt[3]{2}$
$= 4\sqrt[3]{2} + (4)(2)\sqrt[3]{2}$
$= 12\sqrt[3]{2}$

Question:

6. Find the product $\sqrt{3}\left(2\sqrt{6} + \sqrt{3}\right)$.

Solution:

6. $\sqrt{3}\left(2\sqrt{6} + \sqrt{3}\right)$
$= 2\sqrt{18} + \sqrt{9}$
$= 2(3)\sqrt{2} + 3$
$= 6\sqrt{2} + 3$

Question:

7. Simplify $\dfrac{10\sqrt{10}}{5\sqrt{5}}$.

Solution:

7. $\dfrac{10\sqrt{10}}{5\sqrt{5}}$
$= \dfrac{10}{5} \times \sqrt{\dfrac{10}{5}}$
$= 2\sqrt{2}$

(continued)

General and Specific Outcomes
General Outcome Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems. **Specific Outcome** 5.6 Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)

Question:

8. Simplify $\frac{\sqrt{5}+1}{\sqrt{5}-1}$, leaving a whole number in the denominator.

Solution:

8. $\frac{\sqrt{5}+1}{\sqrt{5}-1}$

$$= \frac{(\sqrt{5}+1)(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)}$$

$$= \frac{\sqrt{25}+2\sqrt{5}+1}{(\sqrt{5})^2-(1)^2}$$

$$= \frac{6+2\sqrt{5}}{4}$$

$$= \frac{3+\sqrt{5}}{2}$$

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve questions 1, 2, 4 and 7 • provide an answer to question 6 that may not be in a simplified form; e.g., $2\sqrt{18}+3$.	• solve questions 3, 5 and 8, expressing all answers in simplified form • simplify all radicals in the product, and provide a complete solution to question 6.

STANDARDS IN TRIGONOMETRY

GENERAL OUTCOME

- Solve problems involving triangles, including those found in 3-D and 2-D applications.

SPECIFIC OUTCOMES

6.1 Solve problems involving two right triangles. [CN, PS, V]

6.2 Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V]

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.1 Solve problems involving two right triangles. [CN, PS, V]

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 6.1.
- Students have solved problems involving a single right triangle in Grade 9 Mathematics—Shape and Space (Measurement), specific outcomes 3 and 4.
- Calculators are assumed to be kept running with any multipart calculations. Only final answers are to be rounded. All input data is assumed to be measured. Refer to the Mathematical Conventions Used in the Description of Sample Tasks section of this document.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve problems involving two right triangles, where the shared side is a given value • solve two-dimensional problems, if the diagram is given.	• solve problems involving two right triangles, where the shared side is determined from given information and is needed to solve the second triangle • solve complex problems by drawing his/her own diagram.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.1 Solve problems involving two right triangles. [CN, PS, V]

Sample Tasks

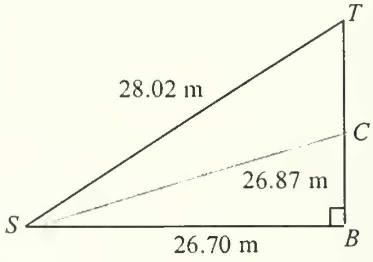
[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. A rugby post has three key points: the base *B*, the crossbar *C* and the top *T*. A surveyor at a point *S* measures the distance *SB* as 26.70 m, the distance *SC* as 26.87 m and the distance *ST* as 28.02 m. From this data, calculate the height of *BC* (crossbar height), to the nearest centimetre, and the angles *CSB* and *CST*, to the nearest hundredth of a degree.

Solution:

1.



$$\begin{aligned} BC^2 + 26.70^2 &= 26.87^2 \\ \therefore\ BC &= 3.02 \text{ m} \end{aligned}$$

$$\begin{aligned} \cos[\angle TSB] &= \frac{26.70}{28.02} \\ \angle TSB &= 17.6567\ldots^\circ \\ \cos[\angle CSB] &= \frac{26.70}{26.87} \\ &= 6.4485\ldots^\circ \end{aligned}$$

Angle *CSB* is 6.45°.

So angle *CST* is
17.6567° – 6.4485°
or 11.21°

The crossbar height is 3.02 m, the angle *CSB* is 6.45°, and the angle *CST* is 11.21°.

(continued)

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.1 Solve problems involving two right triangles. [CN, PS, V]

Sample Tasks

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

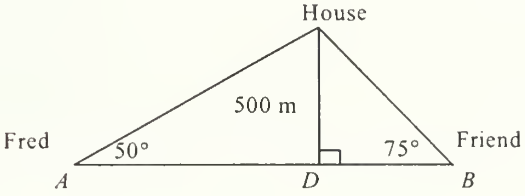
(continued)

Question:

2. Fred and his friend were surveying along a straight road $\overline{AB}$. A house is 500 m from the nearest part of the road. Fred measures the angle to the house to be 50°, while his friend measures the angle to the house to be 75°. How far apart, to the nearest metre, are Fred and his friend?

Solution:

2.



$$\tan 50° = \frac{500}{AD} \qquad AD = 419.55$$

$$\tan 75° = \frac{500}{DB} \qquad DB = 133.97$$

$$AB = 553.52 \text{ m} = 554 \text{ m.}$$

Fred and his friend are 554 m apart.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• calculate the crossbar height and $\angle CSB$ in question 1, if the diagram is given • solve question 2, if the diagram is given.	• solve both questions, even if the diagrams are not given.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.2 Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 6.2.
- This specific outcome requires less time compared to specific outcomes 6.1 and 6.3.
- Students should be able to use a calculator for angles up to 180°.
- Students should be able to explain informally why the sine is positive and the cosine negative for angles between 90° and 180°.
- Exact values for sines and cosines, and the formal treatment of the unit circle, are part of Pure Mathematics 30.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• use a calculator to find the values of the sine and cosine of angles from 0° to 180° • use a calculator to find the angle between 0° and 180°, given the value for the cosine of the angle • use a calculator to find the angle between 0° and 90°, given the value for the sine of the angle • draw the graphs of sine and cosine, when calculator windows are given.	• illustrate the use of sine and cosine ratios in a triangle with one angle between 90° and 180° • set the calculator windows for sine and cosine graphs.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.2 Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

Question:

1. Use your calculator to determine cos 35°, sin 145° and cos 110°, to four decimal places.

Solution:

1. 0.8191, 0.5736, –0.3420

Question:

2. Determine angle A for the following.
   a. $\cos A = 0.47$
   b. $\cos A = -\frac{4}{5}$
   c. $\sin A = 1$
   d. $\sin A = 0.5$

Solution:

2. a. 61.97° b. 143.13° c. 90° d. 30° or 150°

Question:

3. For a triangle ABC,
$(AC)^2 = (AB)^2 + (BC)^2 - 2(AB) \times (BC) \cos B$
For a triangle with $AB = BC = 4$ cm, and $\angle B = 120°$, AC is measured as 7.0 cm. What value for cos 120° does this give? How close is this to the calculator value for cos 120°?

Solution:

3. cos 120° = –0.5313, compared to –0.5000 on the calculator

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can: • solve question 1 • solve question 2, parts a, b and c • obtain one solution for question 2, part d.	**A student demonstrating the *standard of excellence* can also:** • find the second solution for question 2, part d • solve question 3.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Notes:

- This specific outcome is common with Applied Mathematics 10 specific outcome 6.3.
- Both sine and cosine laws should be derived, using either numerical examples or algebraic derivations.
- Calculators are assumed to be kept running with any multipart calculations. Only final answers are to be rounded. All input data is assumed to be measured. Refer to the Mathematical Conventions Used in the Description of Sample Tasks section of this document.
- Triangles with side–side–angle data are not part of this outcome, as side–side–angle is not a test for congruence of triangles.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• determine any side of a triangle, given the diagram and using either the sine or cosine law provided • determine any acute angle of a triangle, given the diagram and using the sine law provided • determine any angle of a triangle, given the diagram and using the cosine law provided.	• determine any side or angle of a triangle, using the sine or cosine law, whether or not a diagram is given • use sine and cosine laws to solve problems involving more than one triangle.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

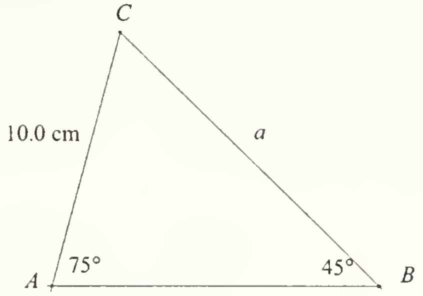
6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. Apply the sine law to solve for side a.

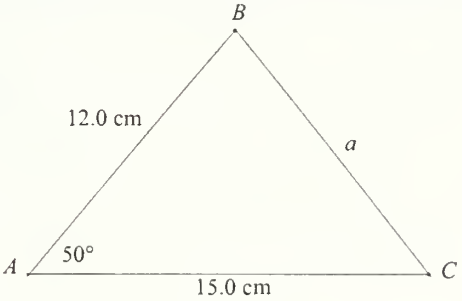


Solution:

1. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ or $\dfrac{a}{\sin 75^\circ} = \dfrac{10}{\sin 45^\circ}$; $a = 13.7$ cm

Question:

2. Apply the cosine law to solve for side a to the nearest 0.1 cm.

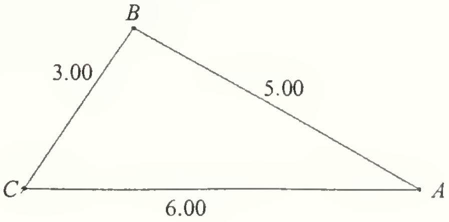


Solution:

2.
$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ a^2 &= (15)^2 + (12)^2 - 2(15)(12)\cos 50^\circ \\ a &= 11.7 \text{ cm} \end{aligned}$$

Question:

3. Solve for angle A to the nearest 0.1°.



(continued)

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

3.

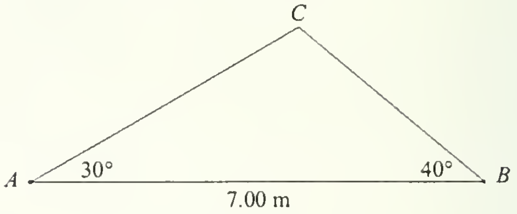
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{6^2 + 5^2 - 3^2}{2(6)(5)}$$

$$A = 29.9°$$

Question:

4. James is building a greenhouse. To take advantage of the sunshine, James constructs the roof as illustrated.



Determine the lengths of the roof sections AC and BC, to the nearest 0.1 m, and determine the angle of the roof, C, to the nearest degree.

Solution:

4.

$$\begin{aligned} C &= 180° - (30° + 40°) \\ &= 110° \end{aligned}$$

$$\frac{7.0}{\sin 110°} = \frac{BC}{\sin 30°}$$

$$BC = 3.7 \text{ m}$$

$$\frac{7.0}{\sin 110°} = \frac{AC}{\sin 40°}$$

$$AC = 4.8 \text{ m}$$

(continued)

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

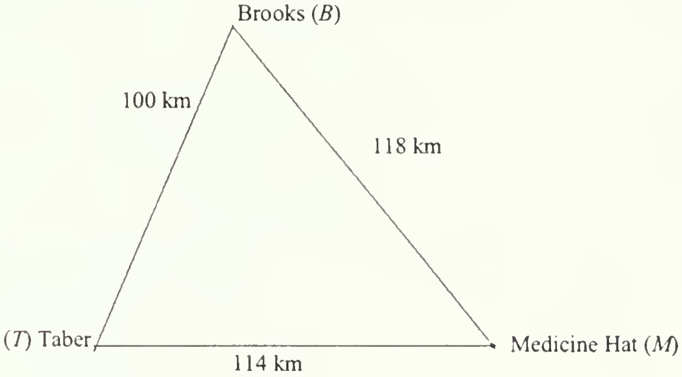
(continued)

Question:

5. Three Alberta towns are situated such that they form a triangle. The distance between Medicine Hat and Brooks is 118 km. The distance between Brooks and Taber is 100 km. The distance between Medicine Hat and Taber is 114 km. Find the smallest angle among the highways.

Solution:

5.



$$\cos B = \frac{118^2 + 100^2 - 114^2}{2(118)(100)}$$

$$B = 62.4°$$

$$\frac{\sin B}{b} = \frac{\sin M}{m}$$

$$\sin M = \frac{m \sin B}{b}$$

$$\sin M = \frac{100 \sin 62.4°}{114}$$

$$M = 51.0°$$

$$T = 180° - (62.4 + 51)$$

$$T = 66.6°$$

The smallest angle is angle M at 51.0°.

(continued)

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

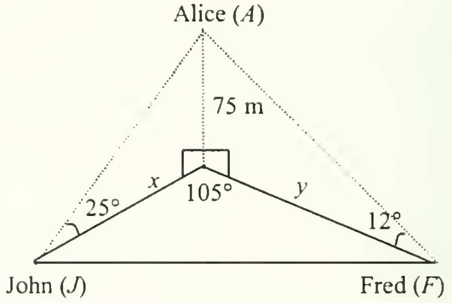
Sample Tasks

☐ Conceptual
☑ Procedural
☑ Problem-solving

(continued)

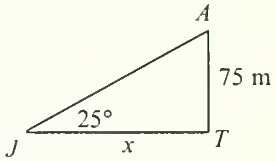
Question:

6. Alice is in a watchtower, and John and Fred are searching for a lost child. The watchtower is 75 m high. Fred radios in that he has found some evidence; he estimates his angle of elevation to the tower to be 12°. Alice radios the information to John. John's estimate of his angle of elevation is 25°. Alice estimates that the angle from Fred to the base of the tower to John is 105°. How far apart, to the nearest metre, are John and Fred?



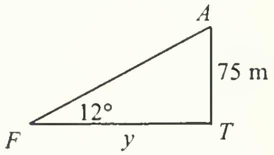
Solution:

6. Let T represent the bottom of the tower.



$$\tan 25^\circ = \frac{75}{x}$$

$$x = 160.838019 \text{ m}$$



$$\tan 12^\circ = \frac{75}{y}$$

$$y = 352.8472582 \text{ m}$$

(continued)

General and Specific Outcomes
General Outcome Solve problems involving triangles, including those found in 3-D and 2-D applications. **Specific Outcome** 6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

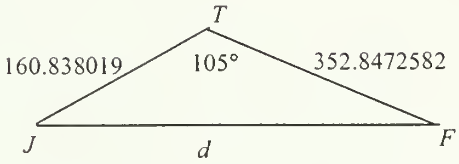
Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

6.



$$d^2 = x^2 + y^2 - 2(x)(y)\cos 105°$$
$$d = 423.965 \text{ m}$$
$$= 424 \text{ m}$$
where $x = 160.838019$
$y = 352.8472582$

Fred and John are 424 m apart.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
• solve questions 1, 2 and 3 • solve questions 4 and 5, if the diagrams are given.	• solve question 5, translating verbal information into diagram form • solve question 6, if the diagram is given.